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COMMON LINES COMPARISON BETWEEN CLARK 1880(ADINDAN- SUDAN DATUM) ELLIPSOID AND (GPS) WGS-1984 ELLIPSOID

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Abstract: *The main objective of this paper is to compare between (world geodetic system) (WGS 1984) and (Adinda- Clark 1880). The comparison figures out the common lines, including, the accuracy and the benefits of each datum's (ellipsoid). The research methodology based on the particular points were being observed by the GPS surveyor receivers and processed using the WGS84 ellipsoid parameters .These points were located on map of Sudan using the ARCGIS (ARC MAP) techniques to get the coordinates and the lines differences and the azimuth comparisons. The comparison results showed negligible difference in the scale and rotation angles between WGS84 and Clarke 1880 ellipsoids.*

Keywords: *Clarhe1880, WGS84, GPS, .ARCGIS.*

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1. INTRODUCTION

Geodesy is the science concerned with the study of the shape and size of the earth in the geometric sense as well as with the form of the equipotential surface of the gravity potential [6]. (Helmert (1880)).The leteral meaning of ‘geodesy’ is ‘dividing the earth ‘, and its first object is to provide an accurate geometrical frame work for the control of topographical and other surveys. The figure of the earth was approximated first by a sphere and later by an ellipsoid . Where as these approximations are of geometrical character, the geoid represents a dynamic reference surface , a certain equipotential surface of the earth’s gravity field [1] . Due to historical developments national departments of survey computed, in the past , ellipsoids best fitted to their country to provide the basis for mapping . Origin and orientation of coordinate system is arbitrary. The national ellipsoids are the geometric reference surfaces only for horizontal coordinates. The world geodetic system (WGS-84) ellipsoid today becomes an important ellipsoid [3]. This is so, because it is the surface on which Global Positioning System (GPS) observations are reduced to. On the other hand, Clarke-1880 is one of the famous traditional reference ellipsoids or geodetic datum. This ellipsoid was widely used in different countries including Sudan.

2. COORDINATE SYSTEMS AND REFERENCE ELLIPSOIDS

2.1 Ellipsoidal geographic coordinates:

As figure1 shows, the earth’s surface may be closely approximated by a rotational ellipsoid with flattened poles (height from the geoid < 100m). As a result geometrically defined ellipsoidal systems are frequently used instead of the spatial Cartesian coordinate system. The rotational ellipsoid is created by rotating the meridian ellipse about its minor axis [2]. The shape of the ellipsoid is therefore described by two geometric parameters, the semi major axis a and the semi minor axis b. Generally, b is replaced by a smaller parameter which is more suitable: the (geometrical) flattening f.

$f = (a-b)/a$.

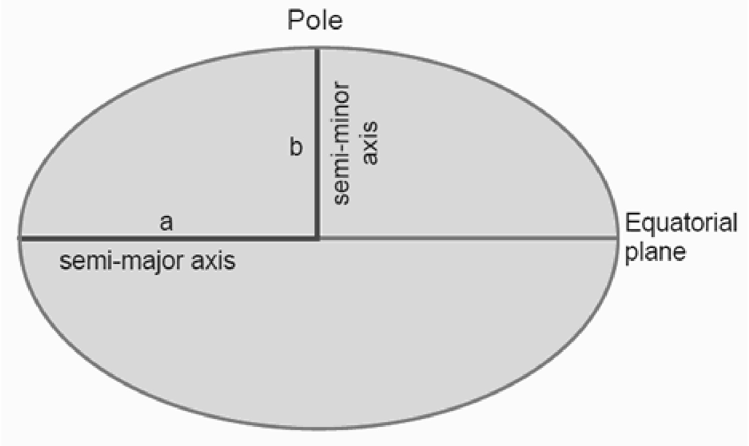

Fig 1. The ellipsoid parameters

The geographic (geodetic) latitude ϕ is the angle measured in the meridian plane between the equatorial (X-Y)-plane and the surface normal at p, where the geographic (geodetic) longitude λ is the angle measured in the equatorial plane between the zero meridian (X-axis) and the meridian plane of p.

2.2 Spatial ellipsoidal coordinate system

For the spatial determination of points on the physical surface of the earth (or in space) with respect to the rotational ellipsoid, the height h above the ellipsoid is introduced in addition to the geographic coordinates ϕ, λ . The ellipsoidal height h is measured along the surface normal(fig.1 5). The spatial ellipsoidal coordinates ϕ, λ, h are designated as geodetic coordinates. The point Q on the ellipsoid is obtained by projecting the surface (or space) point P along the ellipsoidal normal . A point in space is defined by (ϕ, λ, h) and the shape of the ellipsoid (a, f) [7]. A standard earth model as a geodetic reference body should guarantee a good fit to the earth's surface and to the external gravity field, but also, it should posses a simple principle of formation. In this respect, the rotational ellipsoid, already introduced as a geometric reference surface, is well suited. In addition to the semi major axis a and the flattening f as geometric parameters, the total mass M and the rotational angular velocity ω as physical parameters are introduced. The gravity field is then formed as a result of gravitation and rotation [6]. If we now require the surface of this ellipsoid to be a level surface of its own gravity field then, according to Stokes Theorem, the gravity field is uniquely defined in the space exterior to this surface . This body is known as a level (or equipotential) ellipsoid. Additional, the geocentric gravitational constant GM and the dynamic flattening $C_{2,0}$ (2nd order zonal harmonic of an earth gravity model) are given .

If the ellipsoid parameters are given those values which correspond to the real earth, then this yields the optimum approximation to the geometry of the geoid and to the external gravity field: mean earth ellipsoid [5].

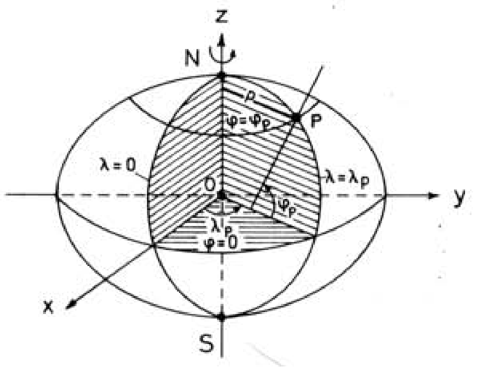

Fig 2: Ellipsoid reference coordinate system

2.3 WGS- 84 ELLIPSOID AS GPS DATUMELLIPSID

2.3.1 Definition of the WGS-84 coordinate system:

The world geodetic system –1984 (WGS-84) coordinate system is a Conventional Terrestrial System (CTS) realized by modifying the Navy Navigation Satellite System (NNSS). or TRANSIT, Doppler Reference Frame (NSWC 9Z-2) in origin and scale, and rotating it to bring its reference meridian into coincidence with the Bureau International de 1’Heure (BIH)-defined zero meridian [5]. Origin and axes of the WGS-84 coordinate system are defined as following:

(a) Origin : Earth’s center of mass.

(b) Z-axis: The direction of the Conventional Terrestrial Pole (CTP) for polar motion, as defined by BIH on the basis of the coordinates adopted for the BIH stations.

(c) X-axis: Intersection of the WGS-84 reference meridian plane and the plane of the CTP’s equator, the reference meridian being the zero meridian defined by the BIH on the basis of the coordinates adopted for the BIH stations.

(d) Y-axis: Completes a right-handed, Earth Centered, Earth fixed (ECEF) orthogonal coordinate system, measured in the plane of the CTP equator, 90 degrees East of the X-axis.

WGS-84 is an earth fixed global reference frame, including an earth model. It is defined by a set of primary and secondary parameters The primary parameters, given in the table below, define the shape of an earth ellipsoid, its angular velocity, and the earth-mass which is included in the ellipsoid of reference.

Table 1: Primary parameters of WGS-84.

PARAMETER	NAME	WGS-84
Semi-major axis	A	6378137 m
Flattening	F	1/298.257223563
Angular velocity	ω	7292115 x10^{-5} rad s^{-1}
Geocentric gravitational constant (Mass of earth's atmosphere included)	GM	398600.5 km^{3} s^{-2}
Normalized 2nd degree zonal harmonic Coefficient of the gravitational potential	$\bar{C}_{20}$	-484.16685 x 10^{-6}

The secondary parameters define a detailed Earth gravity Field Model (EGM) of the degree and order n=m=180. The WGS-84 through n=m=180 is to be used when calculating WGS-84 geoid heights. WGS-84 gravity disturbance components, and WGS-84 $1^{o} \times 1^{o}$ mean gravity anomalies via spherical harmonic expansions. Expansions to this degree and order (n=m=180) are needed to accurately model variations in the earth's gravitational field on or near the earth's surface. The WGS-84 EGM through n=m=41 is more appropriate for satellite orbit calculation (e.g. GPS navigation satellites) and prediction purposes.

2.4 The Clarke 1880 ellipsoid as the ellipsoid used on Adindan Datum

Adindan datum is the historical local datum of Sudan that all triangulation and traverse network observations has subsequently been reduced to it. Adindan base terminal ZY was chosen as the origin of 22º 10' 7.1098" latitude (North) and 31º 29' 21.6079" longitude (East), with azimuth of 58º 14' 28.45" from the north to YY.ZY is now about 10 meters below the surface of Lake Nasser. The Clarke 1880 ellipsoid is used as the mathematical earth shape used for processing the coordinates on Adindan (Sudan) Datum. On the other hand, Clarke 1880 is that ellipsoid of a semi major axis of 6378249.145m, and293.465 reciprocal of the flattening (1/f).

Table 2: The geodetic Clarke 1880 ellipsoid parameters of the Adindan reference system

Parameter	**Symbol**	**Value**
	Defining constants	
Equatorial radius of the Earth	A	6378249.145 m
Semi minor axis (polar radius)	B	6356514.8695 m
first eccentricity	e^2	0. 006803511283
Flattening	F	1 : 293.465

2.5 MERCATOR'S PROJECTION

The Mercator projection is one of the most common cylindrical projections, in which the equator is usually set to be the line of tangency. Meridians are geometrically projected onto the cylindrical surface, and parallels are mathematically projected, producing grate circular angles of 90 degrees. The cylinder is cut along any meridian to produce the final cylindrical projection. The meridians are equally spaced, while the spacing between parallel lines of latitude increases toward the poles. One Characteristic of this projection is conformal - i. e. preserving shapes- and displays true direction along straight lines. Mercator projection is properly the best known of all projections, because it is used for navigation purposes and also in nearly all atlases for maps of the world.

2.5.1 THE TRANSVERSE MERCATOR PROJECTION

The Universal Transverse Mercator (UTM) is simply a transverse Mercator projection to which specific parameters, such as central meridians, have been applied. In other words, the Universal Transverse Mercator (UTM) is a grid-based method of specifying locations on the surface of the Earth that is a practical application of a 2-dimensional Cartesian coordinate system. The UTM divides the surface of Earth between 80°S and 84°N latitude into 60 zones, each 6° of longitude in width and centered over a meridian of longitude. Zone 1 is bounded by longitude 180° to 174° W and is centered on the 177th West meridian. Zone numbering increases in an eastward direction. By using narrow zones of 6° (up to 800 km) in width, and reducing the scale factor along the central meridian by only 0.0004 to 0.9996 (a reduction of

1:2500), the amount of distortion is held below 1 part in 1,000 inside each zone. Distortion of scale increases to 1.0010 at the outer zone boundaries along the equator. In each zone, the scale factor of the central meridian reduces the diameter of the transverse cylinder to produce a secant projection with two standard lines, or lines of true scale, located approximately 180 km on either side of, and approximately parallel to, the central meridian (ArcCos 0.9996 = 1.62° at the Equator). The scale factor is less than 1 inside these lines and greater than 1 outside of these lines, but the overall distortion of scale inside the entire zone is minimized.

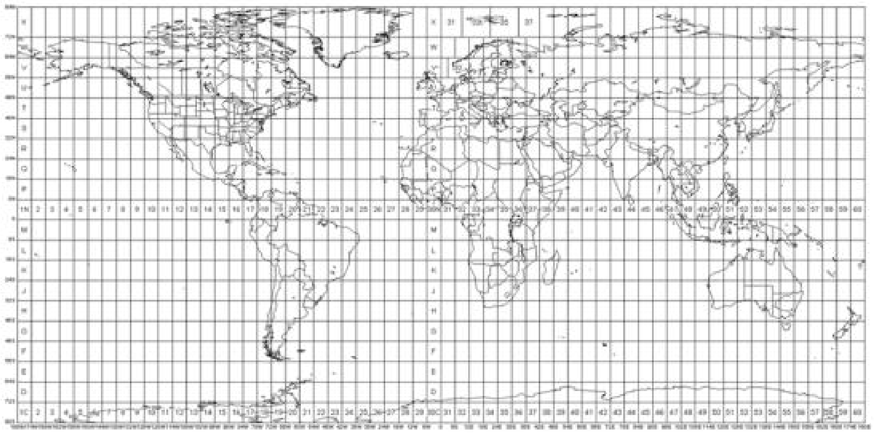

Figure 3: UTM Zones

3. MEASUREMENTS AND RESULTS

3.1 The stations with common coordinates on GPS (WGS84) and Adindan (Sudan) Clarke 1880 ellipsoid.

First of all, points observed as triangulation or Electronic Traverse (ET) distributed over Sudan (Sudan before 2011) were chosen with their coordinates on Adindan datum (Table 3 and Fig. 4), points of class A are first order and those of class B are second order. The same points in table 1 were observed by differential GPS procedure, their coordinates on WGS84 ellipsoid were shown in table 4. Azimuths (bearing) of the lines of the GPS (WGS84) and Adindan (Clarke 1880) points lines were shown in table 3 and table 4 respectively. The comparison of the common lines azimuths (bearings) were shown in table 5 and their standard deviation were shown in table 6. Table 7 shows the standard deviations for the lengths differences of the common lines (WGS84 and Clarke 1880).

4. CONCLUSION

Differences between the UTM projected grid coordinates of the same lines from both WGS-84 and Clarke-1880 ellipsoids are referred to difference in scale and rotation angles of the two ellipsoids . The length differences explain the difference in the scale of the two ellipsoids. The differences in the azimuths of the common lines explain the differences in the magnitude of the rotation angles. The azimuth comparison showed consistency with negligible difference between the two system WGS84 and Clarke 1880, so that we can neglect the rotation angles when we transform from one of them to the other. Also the comparison of the common lines showed an approximate consistent where the largest distance difference is 32 meters, here the difference is large because the two common lines compared are very long indicating that the comparison differences were related to the length of the common lines. Accordingly, it is concluded that, the scale parameter can be neglected from the transformation parameters of WGS84 and Clarke 1880 ellipsoid (on Adindan Sudan datum).

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Table 3: Triangulation and Electronic Traverse station coordinates on Adindan (Sudan) Clarke 1880 ellipsoid coordinates.

points	E (m)	N (m)	ΔE	ΔN	ΔE/ΔN	ATAN	Bearing (degrees)	line
A2	33748	198880	21352	14719	1.450642	0.967254	55.41956	A2-A5
A3	36402	200822	2984	-9249	-0.32263	-0.31209	162.1188	A5-A6
A4	53282	216434	-24336	-5470	4.448995	1.349701	77.33217	A6-A2
A5	55100	213599	16880	15612	1.08122	0.824403	47.23483	A3-A4
A6	58084	204350	6402	-13246	-0.48332	-0.45021	154.2048	A4-A7
A7	59684	203188	-23282	-2366	9.840237	1.46952	84.19732	A7-A3
A8	53112	163034	6572	40154	0.16367	0.162231	9.295178	A8-A7
A9	54034	161106	-5650	-42082	0.134262	0.133464	7.646898	A7-A8
			-922	1928	-0.47822	-0.44607	334.4421	A9-A8
B1	53890	148065						
B2	33122	241820	2817	1753	1.606959	1.014146	58.10628	B13-B1
B3	22278	215072	6765	-6437	-1.05096	-0.81024	133.5768	B1-B15
B4	21316	210644	-9582	4684	-2.04569	-1.11612	296.0509	B15-B13
B5	32331	197219	-11015	13425	-0.82048	-0.68711	320.6317	B5-B4
B6	45713	178731	58126	-48839	-1.19016	-0.872	130.0379	B4-B10
B7	46554	179534	-47111	35414	-1.33029	-0.9262	306.9327	B10-B5
B8	46593	179471	880	740	1.189189	0.871604	49.93922	B6-B8
B9	47388	178601	795	-870	-0.91379	-0.74038	137.5791	B8-B9
B10	79442	161805	-1675	130	-12.8846	-1.49334	274.4379	B9-B6
B11	81780	159179	-15377	40527	-0.37943	-0.36265	339.2219	B14-B7
B12	34159	240891	35226	-20355	-1.73058	-1.04683	120.0211	B7-B11
B13	51073	146312	-19849	-20172	0.983988	0.777328	44.53759	B11-B14
B14	61931	139007	10844	26748	0.405413	0.385164	22.0683	B3-B2
B15	60655	141628	1037	-929	-1.11625	-0.84028	131.8557	B2-B12
			-11881	-25819	0.460165	0.431275	24.71023	B12-B3

Table 4: Coordinates of the same points in table 1 observed by GPS on WGS 84 datum

Points	E(m)	N(m)	ΔE	ΔN	ΔE/ΔN	A TAN	Bearing (degrees)	Lines
A2	33756	198887	21351	14714	1.451067	0.967391	55.42741	**A2-A5**
A3	36409	200826	2809	-9248	-0.30374	-0.29489	163.1043	**A5-A6**
A4	53289	216436	-24160	-5466	4.420051	1.3483	77.25192	**A6-A2**
A5	55107	213601	16880	15610	1.081358	0.824467	47.23849	**A3-A4**
A6	57916	204353	6403	-13245	-0.48343	-0.4503	154.1996	**A4-A7**
A7	59692	203191	-23283	-2365	9.84482	1.469567	84.2	**A7-A3**
A8	53120	163040	6572	40151	0.163682	0.162243	9.295861	**A8-A7**
A9	54041	161113	-5651	-42078	0.134298	0.133499	7.648954	**A7-A9**
			-921	1927	-0.47794	-0.44585	334.4548	**A9-A8**
B1	53898	148073	2817	1753	1.606959	1.014146	58.10628	**B13-B1**
B2	33129	241820	6764	-6438	-1.05064	-0.81009	133.5855	**B1-B15**
B3	22284	215074	-9581	4685	-2.04504	-1.116	296.0581	**B15-B13**
B4	21323	210647	-11015	13425	-0.82048	-0.68711	320.6317	**B5-B4**
B5	32338	197222	58127	-48835	-1.19027	-0.87205	130.0351	**B4-B10**
B6	45720	178737	-47112	35410	-1.33047	-0.92626	306.929	**B10-B5**
B7	46562	179539	880	739	1.190798	0.87227	49.97738	**B6-B8**
B8	46600	179476	795	-870	-0.91379	-0.74038	137.5791	**B8-B9**
B9	47395	178606	-1675	131	-12.7863	-1.49275	274.4719	**B9-B6**
B10	79450	161812	-15377	40523	-0.37946	-0.36268	339.2201	**B14-B7**
B11	81789	159185	35227	-20354	-1.73072	-1.04686	120.0191	**B7-B11**
B12	34165	240890	-19850	-20169	0.984184	0.777427	44.54329	**B11-B14**
B13	51081	146320	10845	26746	0.405481	0.385223	22.07163	**B3-B2**
B14	61939	139016	1036	-930	-1.11398	-0.83926	131.9138	**B2-B12**
B15	60662	141635	-11881	-25816	0.460218	0.431319	24.71276	**B12-B3**

Table 5: WGS84 and Adindan (Sudan) datum on Clarke 1880 ellipsoid Azimuths (Bearings) comparison

Points	Clark bearing (degrees)	WGS bearing (degrees	Difference
A3-A4	**47.23483321**	**47.23849225**	**0.003659038**
A4-A7	**154.2047917**	**154.1995904**	**-0.005201312**
A7-A3	**84.19731896**	**84.20000218**	**0.002683224**
A8-A7	**9.295178353**	**9.295860747**	**0.000682393**
A7-A9	**7.646898232**	**7.648954092**	**0.00205586**
A9-A8	**334.442138**	**334.454766**	**0.012628018**
B1-B15	**58.10627578**	**58.10627578**	**0**
B15-B13	**133.5767952**	**133.5854697**	**0.008674542**
B13-B1	**296.0509214**	**296.0581072**	**0.007185792**
B4-B10	**320.6316638**	**320.6316638**	**0**
B10-B5	**130.037864**	**130.0350672**	**-0.002796753**
B5-B4	**306.9326985**	**306.9290059**	**-0.00369256**
B3-B2	**49.93921554**	**49.9773763**	**0.038160755**
B2-B12	**137.5791475**	**137.5791475**	**0**
B12-B3	**274.4379401**	**274.4719401**	**0.034000053**
B8-B9	**339.2219494**	**339.2200736**	**-0.001875817**
B9-B6	**120.0210501**	**120.0191262**	**-0.001923957**
B6-B8	**44.53758939**	**44.54329277**	**0.005703381**
B7-B11	**22.06829733**	**22.07162886**	**0.003331526**
B11-B14	**131.8556811**	**131.9137956**	**0.058114506**
B14-B7	**24.71023223**	**24.71276064**	**0.002528401**

Table 6:Standard Deviation for Azimuth (Bearing) comparison of the common lines of WGS84 and Clarke 1880 ellipsoid.

Points	Clark Azimuth	WGS Azimuth	Standard Deviation
A3-A4	47.23483321	47.23849225	0.002318406
A4-A7	154.2047917	154.1995904	0.003946808
A7-A3	84.19731896	84.20000218	0.001628401
A8-A7	9.295178353	9.295860747	0.00313934
A7-A9	7.646898232	7.648954092	0.002168152
A9-A8	334.442138	334.454766	0.005307492
B1-B15	58.10627578	58.10627578	0.003738317
B15-B13	133.5767952	133.5854697	0.002395511
B13-B1	296.0509214	296.0581072	0.001342806
B4-B10	320.6316638	320.6316638	0.001529546
B10-B5	130.037864	130.0350672	0.000448057
B5-B4	306.9326985	306.9290059	0.001081488
B3-B2	49.93921554	49.9773763	0.009975263
B2-B12	137.5791475	137.5791475	0.017008465
B12-B3	274.4379401	274.4719401	0.007033203
B8-B9	339.2219494	339.2200736	0.001775087
B9-B6	120.0210501	120.0191262	0.001809128
B6-B8	44.53758939	44.54329277	0.003584215
B7-B11	22.06829733	22.07162886	0.012723174
B11-B14	131.8556811	131.9137956	0.026014243
B14-B7	24.71023223	24.71276064	0.013291069

Table 7: Standard Deviations For common lines Lengths (WGS84 and Clarke 1880) comparison.

Points	Length (m) WGS 84	Length (m) Clark 1880	Difference	Standard Deviation
A3-A4	229919	229929	10	1.885618083
A4-A7	147116	147127	11	2.592724864
A7-A3	234028	234027	1	4.478342948
A8-A7	406850	406880	30	6.12825877
A7-A9	424559	424591	32	7.542472333
A9-A8	21364	21366	2	13.6707311
B1-B15	93379	93381	2	0.471404521
B15-B13	106651	106651	0	0.942809042
B13-B1	33178	33180	2	0.471404521
B4-B10	759186	759201	15	3.299831646
B10-B5	589359	589368	9	0.942809042
B5-B4	173653	173660	7	2.357022604
B3-B2	288605	288624	19	1.649915823
B2-B12	13921	13924	3	9.663792676
B12-B3	284186	284214	28	8.013876853
B8-B9	11779	11780	1	0
B9-B6	16797	16798	1	0
B6-B8	11492	11493	1	0
B7-B11	406839	406839	0	7.542472333
B11-B14	282989	282995	6	3.299831646
B14-B7	433425	433451	26	10.84230398

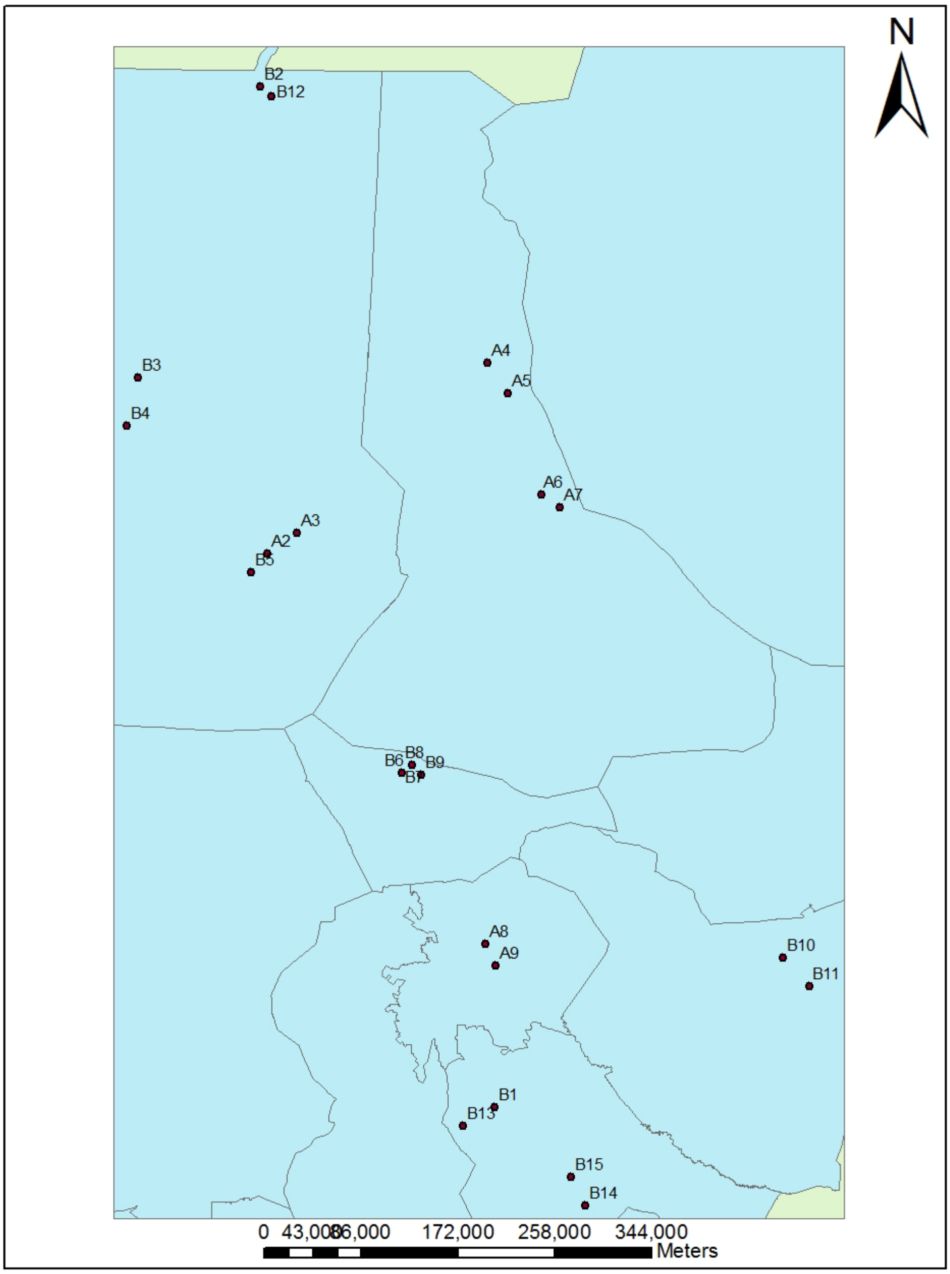

Figure 4 : Show Points in (Sudan Map) Clark 1880 datum (just to show the general position of the points)