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GEOID UNDULATIONS OF SUDAN USING ORTHOMETRIC HEIGHTS COMPARED WITH THE EGM96 ANG EGM2008

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Abstract: *Positioning by satellite system determine the normal height above the ellipsoid, then the direct height observed by the Global Positioning System (GPS) is the height of the point on the surface of the earth measured normal to WGS 84 ellipsoid. This height is not the same as the vertical height of the point to the geoid where the surface of the geoid is approximated to the Mean Sea surface. The main objective of this paper is to test the International geoid models (EGM96 and EGM2008)available by comparison with Orthometric (the local MSL). The geoid undulations determined by each of the mentioned models were tested to serve as the base for the geometrical model.*

Keywords: *Mean Sea Level, Geoid model, Orthometric height, geometric model, ellipsoid.*

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1. INTRODUCTION

The Global Positioning System (GPS) provides X, Y, Z Cartesian Coordinates in a geometric framework called the International Terrestrial Reference Frame (ITRF). The origin (0, 0, 0) 0f which, is the centre of mass of the earth. These Cartesian coordinates are then transformed into ellipsoidal latitude, longitude and height. However the error sources in GPS and the inconsistencies between reference datum of the system and physical reality more or less disturb its performance as a geodetic measurement technique. Today, numerous scientific studies are carried on to develop applicable solutions to eliminate or minimize the weakness of this very practical and precise positioning techniques. Because OPS derived heights refer to its reference ellipsoid WGS84 the height referenced to geoid are required. To take full advantage of the technique in geodetic practical applications, GPS derived ellipsoid heights need to be transformed to orthometric heights via the simple well known equation ($N = h - H$), Where N is the ellipsoid-geoid separation , H is the orthometric height and h is the ellipsoid height). Ellipsoid height is purely geometric value whereas orthometric height has a physical meaning and depends upon the gravity field of the earth. Theoretically science the ellipsoid height and orthometric heights are measured along the normal to ellipsoid and along the direction of plumb line respectively the relation ship defined in the equation above is only an approximation and it serve for most of engineering applications. Also it should be pointed out that these geoid heights determined from global models and gravity data, refer to local vertical datum, due to the sea-surface topography at the reference tide gauge. The local geoidal height at a particular point can be measured by subtracting orthometric height, determined from spirit leveling, from ellipsoidal height determined by GPS observation at the same point. Now the problem remains how to determine N for the points which are not connected by the leveling when observed by GPS.

2. GEOID

The geoid is an equipotential surface corresponding with Mean Sea Level (MSL). The geoid does not exactly correspond to the real ocean surface, because the oceans are subject to tides and current. As the geoid is an equipotential surface, the gravitational potential at any point on it will be the same, and the direction of the gravity at any point on it will be perpendicular to the geoid. If the earth was of uniform density and the earth's topography did not exist, the geoid would have the shape of an oblate ellipsoid centered on the earth center of mass.

Unfortunately the situation is not simple [5]. The geoid is a fundamental physical reference surface. Its shape reflects the distribution of mass inside the earth.

2.1 GEOID AND ITS WAYS OF DETERMINATION

The geoid is a dynamic surface, with its radius vector from the center of gravity of the earth changing cyclically, due to the gravitational attraction of the sun and the moon. This change is of the order of 1 meter [3]. Several geodesists have investigated means of working without the geoid [1]; [2]. Using the classical observations to terrestrial targets, this still appears to be an impossibility, due to the uncertain effects of atmospheric refraction upon vertical angles, and the fact that these are also affected by deflections of the vertical. One technique that is independent of the geoid is geometric satellite geodesy. However it is not imaginable that satellite observations will be made at all geodetic stations, and the classical observations will be complimented by, rather than replaced by, observations to satellites. Although there is only one equipotential surface that may be called the geoid, there are several different ways in which geoidal heights may be computed. This has resulted in several types of geoid, which are briefly described below.

2.1.1 THE SATELLITE GEOID

The satellite geoid is based upon the analysis of orbit perturbation of artificial earth satellites. This representation has the characteristics that, although of uniform quantity, it is a some what smoothed version of the geoid, referred to a geocentric ellipsoid.

2.1.2 THE GRAVIMETRIC GEOID

The calculations of the gravimetric geoid uses the magnitude of the earth gravity, measured at the terrain, to obtain geoidal heights. It is usually referred to geocentric ellipsoid and due to lake of gravity data in certain areas of the world is not of consistent quality.

2.1.3 THE SATELLITE-GRAVIMETRIC GEOID

The combined satellite-gravimetric geoid combines the best features of the satellite geoid and the gravimetric geoid. The detailed variations in geoidal height are described using the gravity anomalies and the large scale variations by using the satellite data [4].

2.1.4 THE ASTRO-GEODETIC GEOID

The calculation of the astro-geodetic geoid uses the direction of gravity (rather than the magnitude) to obtain geoidal heights relative to reference ellipsoid to which the direction is related. This reference ellipsoid is not necessarily geocentric. Due to the nature of the observations it can only be computed for the land masses, and requires a good distribution of data.

2.1.5 THE ASTRO-GRAVIMETRIC GEOID

The astro-gravimetric geoid combines the best features of the gravimetric geoid and the asro-geodetic geoid. It is basically an astro-geodetic geoidwith supplementary deflections obtained via gravity anomalies.

3. THE RELATION BETWEEN THE ELLIPSOID HEIGHT AND THE ORTHOMERTRIC HEIGHT

It is well known that the geoid has no simple mathematic expression, so it is convenient to define the relative simple mathematical surface that closely approximate the actual geoied, thus simplifying the computations. This approximation can be made for a local area or on a global scale. Because it rotates, the earth assumes the shape of a sphere that is flattened at the poles and bulging at the equator. This figure is represented well by an ellipsoid of revolution formed by rotating an ellipse around its minor axis. The ellipsoid is defined by the specification of two parameters, for example, the length of the semi-major axis a and the flattening f. The flattening is related to the semi-minor axis b by: $f = (a-b)/a$ the reason for using the ellipsoid is to obtain a simple surface that approximates the geoid. The relationship between the ellipsoidal height and the orthometric height is $h = H+N$ where N is the geoid undulation with respect to the specific ellipsoid. The geoid undulation is positive if the geoid is above the ellipsoid. A desirable condition for the best fitting ellipsoid is that the magnitude of the geoid undulation be minimized. Thus minimizing the sum of the squares of the geoid undulations distributed over the whole earth ensure a balance of positive and negative undulations. The resulting ellipsoid is called a globally best fitting ellipsoid. In special cases the minimization can be applied to undulations of a small area, the size of a country or continent. This would yield a best fitting local ellipsoid.

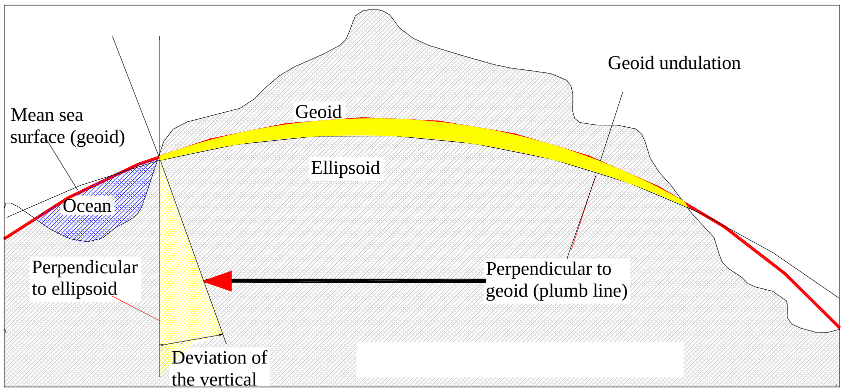

Fig.1: The Earth as geoid

4. EARTH GRAVITATIONAL MODEL (EGM96)

The NGA/NASA Geoid Height File consists of a 15-minute grid of point values in the tide-free system, using the EGM96 Geopotential Model to degree and order 360. The WGS 84 constants used to define the geometry and the normal field of the reference ellipsoid in the calculation of this geoid height file are the following:

i) a=6378137.0 m (semi-major axis of WGS 84 Ellipsoid)

ii) f=1.0/298.2572235630 (flattening of WGS 84 Ellipsoid)

iii) GM=0.3986004418D15 m**3/s**2 (Earth's Gravitational Constant w/ atmosphere)

iv) ω =7292115.D-11 radians/sec (Earth's angular velocity)

The geoid undulation values are calculated by applying a correction term that converts a pseudo-height anomaly calculated at a point on the ellipsoid to a geoid undulation value. In addition, a correction term of -0.53 m is added to the prior result to obtain the geoid undulation with respect to the WGS 84 ellipsoid. The value of -0.53 m is based on the following estimates of the equatorial radius (a) and flattening (f) of an ideal earth ellipsoid in the tide-free system: a=6378136.46 m and 1/f=298.25765. The 15-minute geoid height grid is used as input to a FORTRAN program, INTPT.F, developed for interpolating from the grid, a geoid undulation at any given WGS 84 latitude and longitude. The program F477.F requires both the EGM96 spherical harmonic coefficient file and the correction coefficient file to calculate point geoid undulations at any given WGS 84 latitude and longitude by spherical harmonic synthesis.

5. EARTH GEOID MODEL (EGM2008 - WGS 84 VERSION)

The official Earth Gravitational Model EGM2008 has been publicly released by the U.S. National Geospatial-Intelligence Agency (NGA) EGM Development Team. This gravitational model is complete to spherical harmonic degree and order 2159, and contains additional coefficients extending to degree 2190 and order 2159. Full access to the model's coefficients and other descriptive files with additional details about EGM2008 are provided here in. Those wishing to use EGM2008 to compute geoid undulation values with respect to WGS 84, may do so using the self-contained suite of coefficient files, FORTRAN software, and pre-computed geoid grids provided on the web page. For other applications, the previous release of the full 'Geoscience' package for EGM2008 can be accessed through the link at the bottom of the web page. The WGS 84 constants used to define the reference ellipsoid, and the associated normal gravity field, to which the geoid undulations are referenced are:

i) a=6378137.00 m (semi-major axis of WGS 84 ellipsoid)

ii) f=1/298.257223563 (flattening of WGS 84 ellipsoid)

iii GM=3.986004418 x 10^{14} m^3s^{-2} (Product of the Earth's mass and the Gravitational Constant)

1v) ω=7292115 x 10^{-11} radians/sec (Earth's angular velocity)

All synthesis software, coefficients, and pre-computed geoid grids assume a Tide Free system, as far as permanent tide is concerned. Note that the harmonic synthesis software provided applies a constant, zero-degree term of -41 cm to all geoid undulations computed using EGM2008 with the height_anomaly-to-geoid_undulation correction model. Similarly, all pre-computed geoid undulations incorporate this constant zero-degree term. This term converts geoid undulations that are intrinsically referenced to an ideal mean-earth ellipsoid into undulations that are referenced to WGS 84. The value of -41 cm derives from a mean-earth ellipsoid for which the estimated parameters in the Tide Free system are: a=6378136.58 m and 1/f=298.257686.

6. RESULTS

The main data of this paper based on coordinates of 41 points observed by GPS Surveying receivers (using geodetic observation systems) on WGS84 ellipsoid (Table 1). Each point in table 1 has its own ellipsoid and orthometric height . The geoid separations were obtained by the Earth Geopotential Model 1996 and 2008 (EGM96 and EGM2008). Table 2 shows ellipsoid coordinates with the geoid separation computed using orthometric heights and also using again the EGM96 and EGM2008. Differences between geoid undulations (Orhtometric – GEM96) and (Orhtometric – GEM2008) have been shown in table 2. Figures 2, 3 and 4 show contours of the Sudan geoid undulations using Orthometric heights, EGM96 and EGM2008 models respectively.

7. CONCLUSIONS AND RECOMMENDATIONS

Table 1, shows 42 points distributed over the area of the country (Sudan), positioned by geodetic GPS system on WGS84 ellipsoid, so that the WGS84 ellipsoid height and the elevation (orthometric height) are common to theses points. In table 2 geoid undulations relative to WGS84 ellipsoid height as a reference were obtained using orthometricheight, EGM96 and EGM2008 geoid models. Geoid undulations differences {(orthometric height undulations – EGM96 undulations) and (orthometric height undulations – EGM2008 undulations)} were shown in table 2. The overall general looking to the differences in table two showed that there is no significant differences geoid undulations of EGM96 and EGM2008 compared to the elevation (orthometric height). The average of the differences of EGM96 to orthometric undulations is equal to 1.179822 meters while EGM 2008 to orthometric undulations is equal to 0.531383 meters. According to the average values of the two models (EGM96 and EGM2008) it is concluded that EGM2008 values are closed to the height of elevation (orthometric height) so that EGM 2008 is recommended to be used for determination of the reference height for leveling in the areas which have no reference elevation (orthometric height) in Sudan.

8. ACKNOWLEGMENT

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Table (1): WGS84 (GPS) coordinate with their ellipsoid and orthometric heights

	Latitude dig. Min. Sec.	longitude dig. Min. Sec.	Ellipsoid Height (m)	Orthometric Height (m)
1	17 59 32.934998	31 27 54.383887	359.3949	353.00
2	18 10 10.496701	31 42 52.145941	325.7317	319.30
3	19 35 6.913378	33 18 49.575779	368.5973	361.24
4	19 19 42.752132	33 29 11.288056	522.7519	515.90
5	18 29 28.921023	33 45 0.669784	409.3908	403.40
6	18 23 8.105590	33 55 4.523637	407.0041	401.30
7	14 45 21.395566	33 17 24.234578	403.4121	400.00
8	14 34 53.220220	33 22 31.483305	406.6124	403.00
9	13 24 6.254346	33 21 36.922346	475.6019	473.00
10	21 52 20.547455	31 21 59.451795	337.4061	327.50
11	19 26 35.336135	30 21 33.371423	306.2209	298.30
12	19 2 30.829160	30 16 27.620929	381.9369	373.50
13	17 50 27.136189	31 19 57.459552	360.9299	355.50
14	16 10 32.295922	32 35 58.276118	483.3360	479.80
15	16 14 53.963890	32 40 41.382692	552.7296	549.30
16	16 14 33.513467	32 40 54.354368	410.6173	407.32
17	16 9 50.770628	32 45 22.585886	437.2637	434.04
18	14 37 44.108464	35 44 6.241140	522.6840	523.37
19	14 23 20.350000	35 56 56.054621	573.1138	578.20
20	21 47 21.609669	31 28 3.773009	302.9039	290.90
21	13 14 36.126770	33 5 59.593503	500.6076	501.90
22	12 34 48.880116	34 5 58.653640	447.5745	450.06
23	12 49 3.751063	33 58 58.721626	438.7897	441.22
24	13 34 34.410	22 21 20.223	1040.0834	1031.9
25	15 29 21.214	36 25 36.616	574.4835	571.81
26	11 06 32.616	29 45 08.918	920.4123	921.01
27	18 32 13.1093	31 49 39.293	361.4552	354.9
28	13 03 24.061	30 20 55.902	785.2547	783.70
29	15 36 39.1740	32 32 17.8840	411.0273	410.06
30	21 11 19.05410	30 40 30.43410	197.525	188.1627
31	20 20 13.63744	30 34 19.91158	241.306	232.1627
32	19 42 13.59736	30 23 54.68235	230.047	221.4389
33	18 55 13.43177	33 31 05.88341	346.152	339.7508
34	19 29 43.20861	33 08 52.39356	325.509	318.1891
35	17 44 11.71484	33 59 07.39040	353.454	348.9183
36	16 09 19.37499	32 33 01.89511	387.074	383.8159
37	15 17 56.72386	32 26 49.57072	385.309	382.9513
38	15 35 50.02975	32 36 23.70486	386.975	384.3374
39	14 16 20.75905	36 30 38.29566	572.562	574.0622
40	13 42 54.63437	36 13 10.81828	565.960	568.3148
41	15 01 48.15647	35 56 00.42438	464.436	464.8682

Table (2): The same table 1 coordinates with their Geoid Ellipsoid (WGS84) separations using orthometric, EGM96 and EGM2008

WGS 84 Ellipsoid coordinates			Dimensions in meters				
	Latitude	Longitude	Geoid Ellipsoid (WGS84) separations				
No.	dig. Min. Sec.	dig. Min. Sec.	(Ellip.-Orth.)	(Ellip.-EGM96)	(Ellip-EGM2008)	(Ellip.-Orth.) -EGM96	(Ellip.-Orhto)-EGM2008
1	17 59 32.934998	31 27 54.383887	6.3949	5.9469	5.956	0.4480	0.4389
2	18 10 10.496701	31 42 52.145941	6.4317	5.9977	5.9962	0.4340	0.4355
3	19 35 6.913378	33 18 49.575779	7.3573	7.4953	7.2399	-0.1380	0.1174
4	19 19 42.752132	33 29 11.288056	6.8519	5.7619	6.5781	1.0900	0.2738
5	18 29 28.921023	33 45 0.669784	5.9908	6.1818	5.2011	-0.1910	0.7897
6	18 23 8.105590	33 55 4.523637	5.7041	4.7761	4.9075	0.9280	0.7966
7	14 45 21.395566	33 17 24.234578	3.4121	0.0341	0.3877	3.3780	3.0244
8	14 34 53.220220	33 22 31.483305	3.6124	-0.2766	-0.1531	3.8890	3.7655
9	13 24 6.254346	33 21 36.922346	2.6019	-2.2961	-1.7683	4.8980	4.3702
10	21 52 20.547455	31 21 59.451795	9.9061	9.8431	10.385	0.0630	-0.4789
11	19 26 35.336135	30 21 33.371423	7.9209	8.4689	8.2347	-0.5480	-0.3138
12	19 2 30.829160	30 16 27.620929	8.4369	7.8529	7.4851	0.5840	0.9518
13	17 50 27.136189	31 19 57.459552	5.4299	5.9029	5.7797	-0.4730	-0.3498
14	16 10 32.295922	32 35 58.276118	3.536	-3.3800	2.9811	6.9160	0.5549
15	16 14 53.963890	32 40 41.382692	3.4296	-3.3684	2.9811	6.7980	0.4485
16	16 14 33.513467	32 40 54.354368	3.2973	2.6233	2.9811	0.6740	0.3162
17	16 9 50.770628	32 45 22.585886	3.2237	2.5097	3.0368	0.7140	0.1869
18	14 37 44.108464	35 44 6.241140	-0.686	-0.9080	-1.0286	0.2220	0.3426
19	14 23 20.350000	35 56 56.054621	-5.0862	-1.6242	-1.2982	-3.4620	-3.788
20	21 47 21.609669	31 28 3.773009	12.0039	9.8289	10.3954	2.1750	1.6085
21	13 14 36.126770	33 5 59.593503	-1.2924	-2.1714	-1.9001	0.8790	0.6077
22	12 34 48.880116	34 5 58.653640	-2.4855	-4.2375	-3.8219	1.7520	1.3364
23	12 49 3.751063	33 58 58.721626	-2.4303	-3.7053	-3.1158	1.2750	0.6855
24	13 34 34.410	22 21 20.223	8.1834	8.6234	8.3951	-0.4400	-0.2117
25	15 29 21.214	36 25 36.616	2.6735	-0.8055	-0.3752	3.4790	3.0487
26	11 06 32.616	29 45 08.918	-0.5977	-1.7247	-1.7325	1.1270	1.1348
27	18 32 13.1093	31 49 39.293	6.5552	6.6302	6.5647	-0.0750	-0.0095
28	13 03 24.061	30 20 55.902	1.5547	0.3907	0.4034	1.1640	1.1513
29	15 36 39.1740	32 32 17.8840	0.9673	1.9913	2.5827	-1.0240	-1.6154
30	21 11 19.05410	30 40 30.43410	9.3623	9.6610	9.7149	-0.2987	-0.3526
31	20 20 13.63744	30 34 19.91158	9.1433	9.2230	8.9969	-0.0797	0.1464
32	19 42 13.59736	30 23 54.68235	8.6081	8.8330	8.5678	-0.2249	0.0403
33	18 55 13.43177	33 31 05.88341	6.4012	-0.0770	6.0805	6.4782	0.3207
34	19 29 43.20861	33 08 52.39356	7.3199	7.0000	7.0385	0.3199	0.2814
35	17 44 11.71484	33 59 07.39040	4.5357	3.9540	3.9841	0.5817	0.5516
36	16 09 19.37499	32 33 01.89511	3.2581	2.6440	3.1126	0.6141	0.1455
37	15 17 56.72386	32 26 49.57072	2.3577	1.6420	1.9583	0.7157	0.3994
38	15 35 50.02975	32 36 23.70486	2.6376	1.8640	2.537	0.7736	0.1006
39	14 16 20.75905	36 30 38.29566	-1.5002	-2.5570	-0.9377	1.0568	-0.5625
40	13 42 54.63437	36 13 10.81828	-2.3548	-3.8280	-3.0093	1.4732	0.6545
41	15 01 48.15647	35 56 00.42438	-0.4322	-0.8590	-0.8749	0.4268	0.4427
The average						1.179822	0.531383

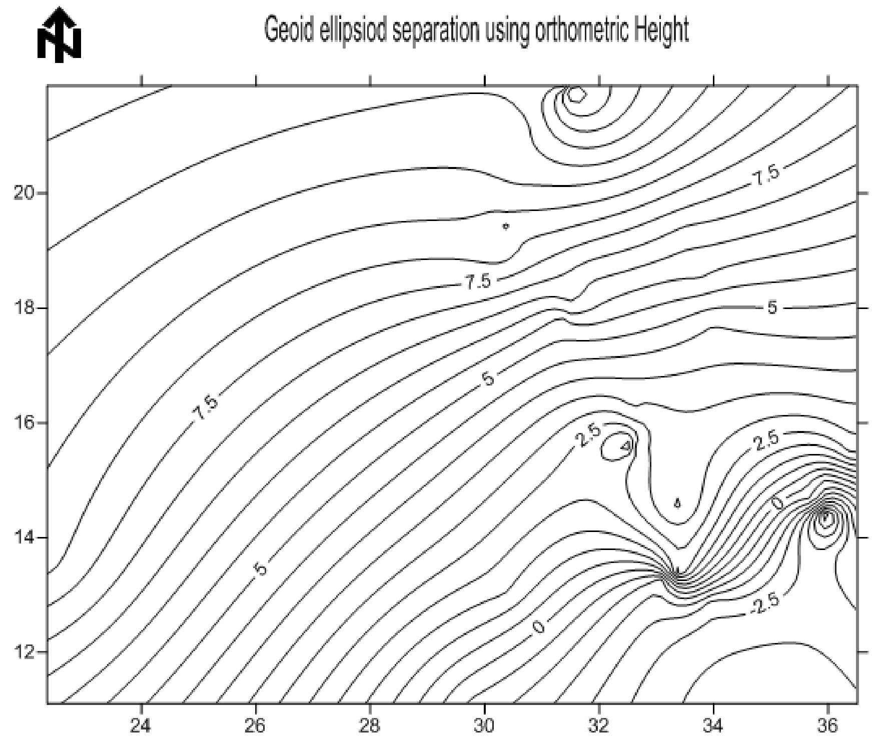

Fig (2): Sudan contour lines of geoid ellipsoid seperations using the elevations (orthometric heights)

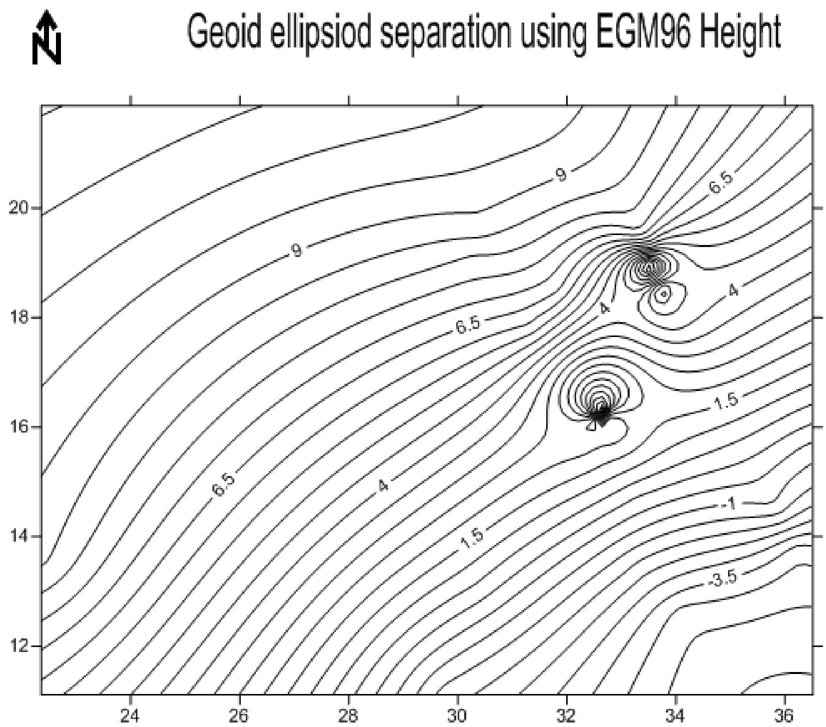

Fig 3: Sudan contour lines of geoid ellipsoid separation using EGM96

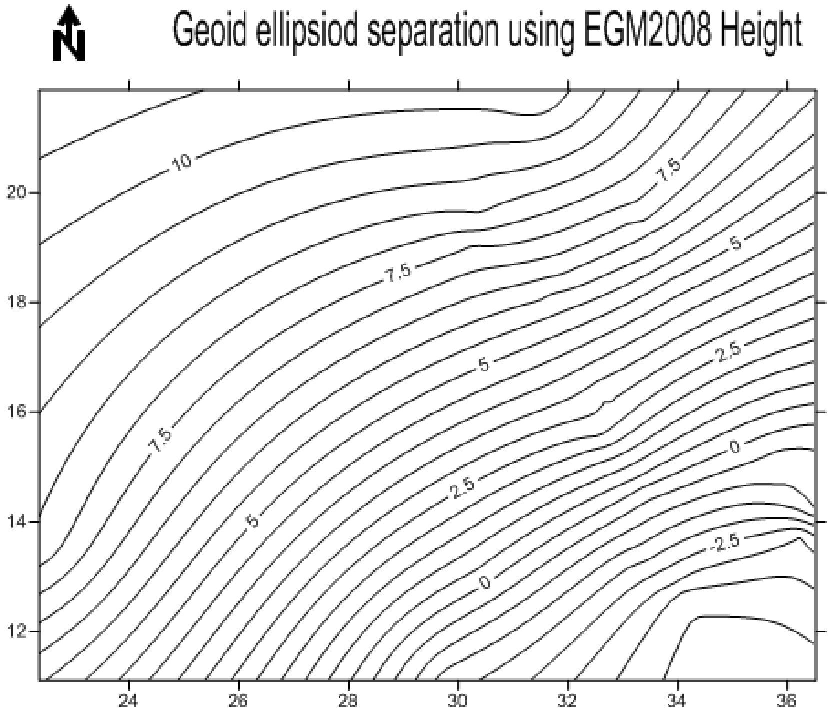

Fig. 4: Sudan contour lines of geoid ellipsoid separation using EGM2008