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FIN GEOM: A PERSONALIZED SYSTEM OF INSTRUCTION ON FINITE GEOMETRY

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ABSTRACT

A Personalized System of Instruction was designed and constructed. The instructional material was divided into two units. The first unit is the introduction on the concept of geometry as an axiomatic system consisting of the following: undefined elements, definitions, axioms and theorems, and illustrations. While the second unit emphasized on the finite geometry which consists of its formal definition, several illustrations, homogeneous coordinates, some theorems with their proofs, and specializing the two dimensional geometry under consideration based on deductive system.

The instructional material began with the introduction and definitions. Each unit contains Self-Assessment Questions or SAQ. Answers to Self-Assessment Questions (SAQ) are located in a separate section for the sake of organization. The instructional material used variety of graphics and illustrations and fancy fonts to emphasize important concepts and to present a pleasing presentation of finite geometry.

KEYWORDS: *Personalized System of Instruction, geometry, Finite geometry, mathematics*

I. INTRODUCTION

Geometry has been a field of great interest as it developed through the years. The subject as we know is the study of section and projection of infinite number of points and lines on a plane.

Today, there are geometries with finite numberof points and finite number of lines on a plane. This is finite geometry where lines contains only finite number of points, and through each of such point pass only finite lines.

Finite geometry provides an important approach to the study of some branches of mathematics, especially projective geometry. It is an inherently simple and elegant geometry similar to high school Euclidean geometry. The knowledge of finite geometry prepares a student for higher geometry. It helps the student in establishing proper logical sequence, recognizing the undefined terms, specifyingappropriate axioms and distinguishing the validity and invalidity of axioms and theorems or geometric figures.

The existence of many literatures on the basic concepts of finite geometry is abundant. In addition, present day understanding of finite geometric structure requires a successive learning in mathematics, each level should be clearly understood, otherwise, everything from them on is in peril. Thus, the idea of personalized instruction of material would be helpful in order to guide the student in every phase.

The Personalized System of Instruction or PSI is an innovative devised to overcome the difficulties in learning in the absence of instructor/educators to answer student's query. In addition, when it is difficult for an educator to clarify a point or correct himself, it is essential to provide material that can guide the student in every step and anticipate most information requirements. Furthermore, an instructional material, or PSI requires a level of detailed subject analysis that is better than conventional classroom situations.

STATEMENTOF THE PROBLEM

This study aims to develop an instructional material in the form of a printed study guide, known as Personalized System of Instruction (PSI) on finite geometry. The material should be able to present and illustrate finite geometric concepts in its simplest and comprehensive lesson design.

Significance of the Study

In the field of mathematics, many concepts are quite difficult to understand, so, there is a continuous search for approaches and methods that would expand students' learning process. Some studies support the use of individualized system of instruction in helping students perform effectively.

Personalized System of Instruction or PSI is an alternative for formal education. If the PSI would be fully applied, it would help in upgrading the quality of education. It can also help varied students especially the slow learners.

PSI, in the form of a printed study guide, is widely used in teaching. This material was organized in such a way that learners can do most of their learning from the material.

This Personalized System of Instruction or PSI on finite geometry is very useful for prospective learners especially College freshmen majoring in mathematics. This would help them generalized certain notions of points and lines, examine how geometric structure can serve as model for axiom systems and how homogenous coordinates can be used to select a point with a unique ordered pair ina two dimensional plane. It also makes the learners think which is very crucial in greater pursuit of higher mathematics.

The subject matter is not a new concept, and does not involve algebraic complexity like in analytic geometry. Proofs of theorem with the aid of illustrations are relatively elementary in nature.

Using colorful graphics, different colors and fancy fonts would give the material attractive.

Finally, the importance of PSI in learning can be seen in terms of the opportunities for the individuals to develop their abilities. This emphasize that the modes of teaching must be parallel by an attempt to give an individual an opportunity to his or her learning.

OBJECTIVE OF THE STUDY

The main objective of the study is to provide an instructional module known as Personalized System of Instruction or PSI that will support the student in understanding and applying the basic concepts of finite geometry.

Specifically, the author aim to:

1. Design an instructional material that can motivate or stimulate the interest of the prospective learners to carry on with their learning.
2. Explain the basic concepts of finite geometry in an illustrative manner.
3. Help and guide the student to progress in their studies and move toward better understanding of the lesson.

Time and Place of the Study

The study was conducted at the College of Teacher Education, during the first semester of Academic Year 2015-2016.

Scope and Limitation

This study will cover only the following topics:

1. Geometric structure and finite geometries
2. The elements of axiomatic system
3. Geometry of seven points and seven lines
4. Homogenous and non homogeneous coordinates
5. Finite geometries of two-dimensional plane

The choice of finite geometry is quite personal as well as in consideration of the Modern Geometry course. Finite geometry is a non-Euclidean geometry and is preparatory topic before projective geometry.

The design and construction of the module could have been better in its presentation and illustrations if there were no boundaries, such as limited time for research and blending of ideas for different references, limited resources from which we could have managed the presentation and reproduction of the module.

Moreover, the author was not able to pre-test the draft module to college student for further improvement, due to time constraint.

Another factor that the author considered was the limited experience in doing PSI.

REVIEW OF LITERATURE

Projective geometry started as early as the dawn of Renaissance in the famous painters during the middle Ages and those who contributed to this ideas and development were artists rather than scientists and engineers. These great advances in the mathematical research on projective geo metrywas attributed to the pioneering works of some mathematicians like Duccio (1255-1314), Gerard Desargues (1593-1662), Jean Victor Poncelet (1788-1867), August Ferdinand Moebius (1790-1868), Arthur Cayley (1821-1895), Felix Klein (1849-1925), and David Hilbert (1862-1943), to name some few.(Constantino D Leonor, Projective geometry, vol.1. of Lectures in Mathematics)

In 1889, Gino Fano broadens the idea of a finite geometry of seven points and seven lines. Law tells that Fano extended this to three dimensions that now contains 15 points and 15 lines. Fano laid down several properties and principles concerning this simplest non-trivia finite geometry.

In 1906, Oswald Veblen (1880-1960) and W.H Bussey showed that there exist projective planes containing a finite number of points and lines. They have generalized for arbitrary dimension *n.*They have shown that this plane of order two cannot be drawn using straight Euclidean lines. (Ott, 1937)

Clement Lam described the evolution of the finite geometry of order ten and how computer was used to solve it. He relates the existence of finite plane of order n to the existence of Latin square. Moreover, he uses the computer to create the incidence matrix of order 10 that involves combinations of row and column permutations.

The test incidence matrix of a finite plane, is then the operations of permuting the rows ad permuting the columns of matrix correspond only to reordering the lines and points of the plane. These operations preserve the property of being projective. (Lam, 1991)

Carmichael(1930) in his “Finite Geometries and the Theory of Groups” showed the connection between finite geometry and the theory of groups. He presented a finite geometry PG (k, p^n) by means of Abelian group f order p where p is prime. Using this representation a system of coordinates for denoting the elements of an Abelian group is introduce by means of Galois field. He commented that every finite field is a Galois field. Moreover, they are used to aid the section and definition of a normal set of subgroups that are interpreted as points of a finite plane G (k, p^n) of k dimensions. By using these results, he exhibits the collineation groups of finite geometries.

Minhalek (1972) shows a procedure for coordinating the geometric plane. He starts with the fixing of three non-collinear points called base points, which consists of the origin, and two ideal points, then with fixing of scale on a line. On a set of points on a line, he assign a point to any two given points on the set, independent of the three fixed points, they are called scale or gauge point. Then he defined the arithmetic which entry is within or outside the segment defines by the gauge point or scale point.

Instead of fields, Adrian Albert, Reuben Sandler, and others used algebras such as near fields, semi-fields, quasi-fields, loops, groups, and ternary rings as ground algebras for finite geometry. One of the ground algebra that is of great interest in the subject of projective plane is ternary ring defined by the same plane. Ternary rings as mention in the book is determine by our choice of the ordered set of our points making up the coordinating quadrangle. The choice of the coordinating quadrangle determined the ternary function in terms of the geometry of the plane relative to that quadrangle. Thus, by selecting another ordered set of four points as the coordinate quadrangle, a completely different ternary ring could be obtained. Hence, the used of planar-ternary ring was to introduced the coordinate system of the arbitrary finite plane.

Harold L. Dorwart introduced the Latin square of order q that is a qxq matrix whose entries are integers between one and$q$. In every row and every column of the matrix, no entries are repeated. He remarked that two Latin squares are orthogonal upon if every ordered pair of numbers appears once in theresulting square.

Dorwart also declared that it is possible to prove that if $n=p^{\alpha}$ where p is prime and α is a positive integer, then for every $n \geq 3$ there exists a complete set of $n-1$ orthogonal Latin squares of order n.

In addition, he introduced the technique on perfect difference sets that appears to have a connection on cyclic finite geometry. He defined a perfect difference by using the $n+1$ integers $d_1, d_2, \ldots, d_{n+1}$ forming a set if their $n^2 + n$ differences $d_i - d_j$, where i and j each take all values 1, 2,…, $n+1$ but $i \neq j$, are simply minus or plus the first $(n^2+2)/2$ positive integers in some order. He also remarked that an incidence table for finite plane of order q is evidently very easy to write down when one knows a perfect difference set of order q. Moreover, by reversing the procedure, the incidence table forms a $q-1$ complete orthogonal set of$q$-by-q Latin squares.

II. METHODOLOGY

The requirements for this study were accomplished by the author by doing the following activities:

1. The author studied the Mathematics curriculum to determine the topic for the specific PSI module.
2. The author consulted expert in the field Mathematics for the approval of the topic of PSI material.
3. The author also reviewed the basic concepts and knowledge of the subject discussed with expert in the field of Mathematics.
4. Opinions and suggestions were also sought concerning the topics tobe discussed in the actual paper.
5. The author constructed the outline of the Instructional material.

6. The author consulted some textbooks in geometry and finalized the concepts to be tackled. Articles from journals, magazines and proceedings was also obtained.
7. The author started writing the materials for the PSI.
8. The author submitted the finished instructional material to the expert for revisions.
9. Revisions of the module were made according to the recommendations of the PSI expert.
10. Finally, the author has written the final PSI material.

III. RESULTS AND DICUSSIONS

The order of the entire lesson on finite geometry was presented based on the deductive approach to Geometry. Two units were constructed. These are

Unit 1: Introduction to Finite geometry: Axiomatic System

Unit II: Finite Geometry

Unit I introduced the basic elements of an axiomatic system. These are undefined terms, axiom/postulates, definitions, and theorems.

Unit II dealt with the basic structure of finite geometry as deductive system and presented the homogeneous and non homogeneous coordinates of points and lines in two-dimensional plane. These coordinate systems were used to obtain the other finite geometries.

Each unit consists of the following parts in order:

<u>Introduction</u>

This part was intended to give the general idea of the subject to be covered and to prepare the students for main discussion.

<u>Discussion</u>

This is the lesson proper in which the concepts of the subject are presented. It includes definition of terms, explanations of concepts, illustrations, exercises, and figures. The topic was discussed in the simple and conversional way, since comprehension of the subject matter relies greatly on the discussion.

The discussion itself contains the following:

Definition: constitute the main concept to be discussed: the key concepts, basic ideas, and properties.

Illustrations: the presentation of concept or definitions. It illustrate the concepts to be introduced and proceeded by explanation concerning the example.

Figures: geometric presentation or model of a concept or examples different from illustrations.

Self-Assessment Question: SAQwas constructed to guide the students in analyzing their skills and applying the concept as discussed in the illustrations.

Remarks: this includes notes about the concept discussed.

Answer Keys: In this part, the students are given the chance to discover their own error and measure their knowledge of the subject. Answers to several SAQ were provided at the end of the module for the learners to check their answer.

As part of this research problem which is to construct a PSI. The author was given the privilege to think of his version of an instructional material. In his version, there are several features familiar with the readers.

FEATURES

In terms of presentation, the paper itself has a blue cover page with geometric figures. This cover page will serve to catch the attentions of the readers. The author used this cover to project the title of the PSI materials. The cover also gives the readers a motivation on the presentation and effectiveness of the materials.

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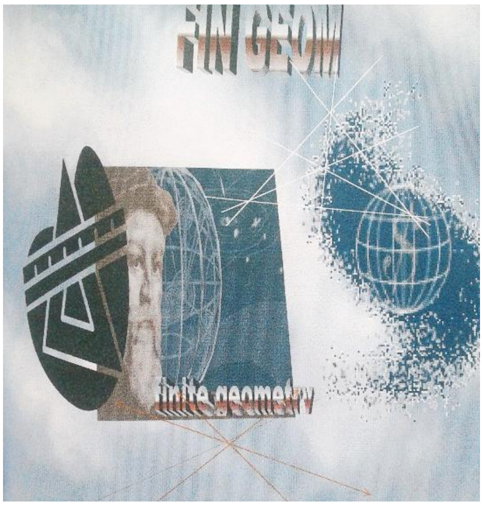

The table of contents gives you an idea about the topics and particular pages of these topics. It guides the students to the flow of the entire lesson.

Prompters

The following prompters were used for the purposed of giving emphasized and as eye catcher. Variations were used to distinguish one lesson to another.

A ***pile of books*** was chosen to represent the lesson proper to indicate that what the learner is reading is either a definition or a concept. Books also emphasize the importance of what the learner is reading.

A ***hand holding a pen*** guide the readers to the following illustrations.

A ***question mark*** prompts the reader to prepare for the self-assessment questions.

A ***ruler, compass, and a triangle*** prepare the reader to prove theorems.

A ***bulb*** provides the answer key to the self-assessment questions.

The heading of each paper was given also emphasized by blue line where finite geometry was written. That simply made the paper a user-friendly material. Thus projecting a clear vision and catches the interest of the reader.

Other graphics were used to introduce or discuss the lessons on each unit.

IV. SUMMARY

The study concerns itself with the construction of Personalized System of Instruction on Finite Geometry an alternative to lecture method. Topics were group into two: namely the axiomatic system and the finite geometry.

The axiomatic systems consists of the following: undefined terms, definitions, axioms and theorems, and illustrations. The finite geometry consists of formal definitions, some illustrations: three points and three lines geometry and seven points and seven lines geometry. Homogeneous coordinates, theorems and proofs were also designed and constructed.

The module was submitted to the expert for revisions.

The module provided lectures, appropriate examples, self-assessment or feed back and modified prompters to attract the readers. Several features were added to meet the objectives of the study.

V. CONCLUSION

PSI is a medium of instruction to resolve the concern of professors for their students. It is a self-pace student -centered teaching and an alternative form of teaching/lecturing. This will be a relief on the part of instructor since it will help them teach effectively with lesser effort in their part. The PSI uses text and other reading materials to present the concepts covered in the discussion. There are many factors to be considered in making PSI some of these are: the subject itself, individual differences, and the combined interest of the student as well as the teachers.

This research is about the construction of instructional materials for college students.

The PSI covered finite geometries. The elements of axiomatic systems and geometric structures were included. Definitions, illustrations, axioms, homogenous coordinates, theorems, and proofs were designed, constructed, revised and evaluated with the help, supervision and evaluation of the expert in mathematics and module writing. The expert in the field checked if:

1. The contents are readable to the students;
2. The design is appropriate to the prospective reader;
3. The term used are understandable and appropriate.

VI. RECOMMENDATIONS

The author would like to recommend the following for the improvement of this paper:

1. to reproduce copies of PSI for the prospective learners.
2. to conduct an actual test of the printed materials to the prospective readers, in order to evaluate the effectiveness and to determine which area needs to be improved.
3. add more graphics, visuals, designs and other colorful features to the PSI to make the material more lively and interesting to read.
4. more self-assessment questions to test students comprehension skills.

BIBLIOGRAPHY

Adler, Clair Fisher (1968). *Modern Geometry.* New York: McGraw-Hill Book Co., 1985.

Albert, A.A. and Sandler, R. *An Introduction to Finite Projective Planes.* New York: Holt, Rinehart and Winston.

Carmichael, R.D (1930). *Finite Geometries and the Theory of Groups.* The American Mathematical Monthly, vol. 52.

Coxeter, H.M. *Projective Geometry.* New York: Blaisdell Publishing Co., 1964.

Coxeter, H.S. M. *The Real Projective Plane.* Cambridge, New York: Cambridge University Press, 1960.

Dorwart, Harold L. *The Geometry of Incidence.* New Jersey: Prentice-Hall, Inc. 1964.

Gleason, Andrew M. *Finite Fano planes.* The American Mathematical Monthly, vol. 41, 1956.

Hartshorne, Roland. *Foundations of Projective Geometry.* New York: Academic Press, 1964.

Levy, Harry. *Projective and Related Geometries.* New York: The MacMillan Company, 1964.

Lam, C.W. H. *The Search for Finite Projective Plane of Order 10.* The American Mathematical Monthly, vol 98, 1991.

Leonor, C. D. *Projective Geometry*, vol. 1 *Lectures in Mathematics.* Philippines: I.M.S.P. University of the Philippines, 2000.

Minhalek, R.J. *Projective Geometry and Algebraic Structures.* New York; Academic Press, 1972.

Ott, E.R. *Finite Projective Geometries, PG (k, p^n),* The American Mathematical Monthly, vol. 44, 1937.

Scott, Foresman. *Geometry.* 2nd ed. Glenview, Illinois Foresman and Company, 1984.

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