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## LINEAR CONTROLLER DESIGN FOR VELOCITY CONTROL OF A DC SERVO-DRIVE

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### **ABSTRACT**

*Linear controllers for position and velocity control have become very popular in systems automation and control of industrial processes. This is easily attributable to their robustness, easy of design and implementation and simplicity. Velocity control of industrial systems is very important for accurate tracking and regulation of systems to minimize error and improve performance. This paper presents a linear controller for the velocity control, of a DC servo driven system. The controller was designed to meet certain performance criteria such as near zero steady state error, small settling and rise times, reduced overshoot and transient response. Simulations were performed using Matlab/Simulink software environment and the results were analyzed. Analysis results showed that the designed linear controller was able to perfectly control the velocity output of the DC servo system.*

**INDEX TERMS:**Linear controller, Velocity control, DC servo, simulations, Lag compensator.

### **I. INTRODUCTION**

Servo-based actuation mechanisms are often used in most industrial concerns. Typical examples of servos are; Pneumatic, Hydraulic and Electrical servos. The Dc electrical servos have gained more popularity than the rest due mainly to the following; low cost, simple to operate and design, and the fact that control of the output position and/or velocity is easy [1], [2], [3]

[4].

Proportional, Integral and Derivative, (PID) based controllers and their variants are simple and easy to design and implement. This coupled with their robustness to uncertainties in model

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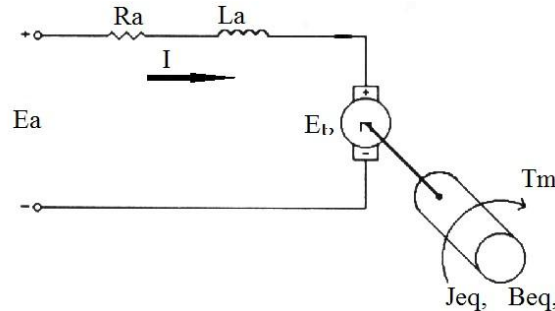


Fig. 1. The schematic representation of the DC-servo motor

Parameters made them popular in industrial automation and systems control engineering environments. System performance such as the rise time, settling time, transient response, steady state error and peak overshoot can greatly be improved by the combinations of the Proportional, Integral, and Derivative control action in a feedback scheme. The general model of the DC servo motor (Permanent Magnet DC type) driven system can be obtained from the schematic shown in figure 1. The model of the servo driven system [5], [6] is given as

$$\frac{\omega_l(s)}{E_a(s)} = \frac{AK_m}{J_{eq}R_a s + (AK_m)^2} \quad (1)$$

This equation can further be reduced to

$$\frac{\omega_l(s)}{E_a(s)} = \frac{1}{\frac{J_{eq}R_a s}{A+K_m} + AK_m} \quad (2) \text{ Given}$$

$$K = \frac{1}{AK_m}$$

and

$$T = \frac{R_a J_{eq}}{(AK_m)^2}$$



then

$$G(s) = \frac{\omega_l}{E_a(s)} = \frac{K}{Ts + 1} \quad (3)$$

where  $E_a$  is the motor armature voltage of the motor,  $J_{eq} = (A^2J_m + J_l)$  is the equivalent inertia,  $J_m$  the motor inertia,  $J_l$  the load inertia,  $K_m$  is Motor torque constant,  $\omega_l$  the angular velocity,  $A$  is the gear transmission ratio and  $R_a$  the armature resistance.

Equation 3 is the open loop transfer function of the linear model structure of the DC servo drive whose output is the velocity and input the armature voltage.

TABLE I

PARAMETERS OF THE FRICTION TEST-BED

Parameter	Description	Value	Unit
$K_m$	Motor torque constant	0.00767	N- m/A
$A$	Gear box transmission ratio	70:1	-
$J_l$	Moment of inertia of load	$2.6583 \times 10^{-5}$	$kgm^2$
$J_m$	Moment of inertia of the motor armature	$3.87 \times 10^{-7}$	$kgm^2$
$K_b$	Back emf constant	0.00767	N- m/A
$R_a$	Armature resistance	2.6	$\omega$

The layout of the paper is thus: In section II, the design of the linear controller meeting certain performance requirements to be used for velocity control implementations is presented. Position control simulations are carried out in section III, while the result of the various control implementations are discussed in section IV. Finally, section V contains the conclusion of the paper.



## II. Lag Compensator Design

The objective is to design a velocity controller for this system to meet certain criteria such as steady state error of 0.01 to a step input signal, and a cross-over frequency of about 110 rad/sec.

### PROCEDURE

A lag compensator structure is chosen for the task of velocity control over the PI controller because it reduces the influence of high frequency noise signals on the overall control scheme. The parameters of the DC servo system that are of interest as contained in the manufacturer's data sheet are shown in table I.

First a check to ascertain if the original system meet design specification without any modifications was performed as follows: Obtain the open loop Bode plots; From this Bode plot of the open loop transfer function  $G(s)$  in figure 2b it is observed that the cross-over frequency criterion is not met since it's cross-over frequency is 65.8 rad/sec.

The closed loop response of the system to a step input signal as presented in figure 2a shows the steady state error 0.35 to be much greater than specified. Introducing a lag compensator

Network [7]

$$C_{lag} = K_p \frac{T_i s + 1}{T_i s + \gamma} \quad (4)$$

with  $K_p$  being the proportional gain,  $T_i$  time constant, and  $\gamma$ .

*Design for the gain parameter:*

Since  $G(s)$  is a first order type 0 system, the steady state error  $e_{ss}$  to a reference step is

$$e_{ss} = \frac{1}{1 + K_n} \quad (5)$$

where  $K_n = KK_p$  is the loop gain for the entire system which ensures the error criterion is satisfied. Substituting values yields  $K_n = 99$  and from there

$$K_p = \frac{K_n}{K} = 53.1401$$



Therefore the loop gain is modified to  $K_p G(s)$ . Plots for the open loop Bode plot and the closed loop response to a step input are then obtained and compared with specifications. From the closed loop response figure 3a, the error requirement of 0.01 is now satisfied. However, from the Bode plot of figure 3b the cross-over frequency of  $4.15 \times 10^3$  rad/sec has become too high. From the open loop Bode plot of figure 2b, it is observed that a magnitude shift of 3.6dB will meet the cross-over frequency criterion. This adjustment thus alters the system gain also. This alteration in the error would be corrected via the lag compensator. Having met the cross-over frequency requirement of 110 rad/sec as shown in figure 4b, a plot of the closed loop step response is obtained and it shows some deviation at steady state from expected, figure 4a. The parameters of the lag network are then computed as;

$$\gamma = \frac{K_p K e_{ss}}{1 - e_{ss}} = 0.0285$$

and

$$T_i = \frac{10}{\omega_c} = 0.0909$$

Where  $\omega_c$  is the specified cross-over frequency. Hence the parameters of the lag compensator for the control of the system are  $K_p = 1.5136$ ,  $\gamma = 0.0285$  and  $T_i = 0.0909$  and equation 4 becomes

$$C_{lag} = 1.5136 \frac{0.0909s + 1}{0.0909s + 0.0285} \quad (6)$$

This equation was used for the velocity control simulations as the closed loop response figure 5a and the Bode plot figure 5b meet design requirements. The parameter values of the linear controller yielding desired performance result is now used for the velocity control simulations in the next section.

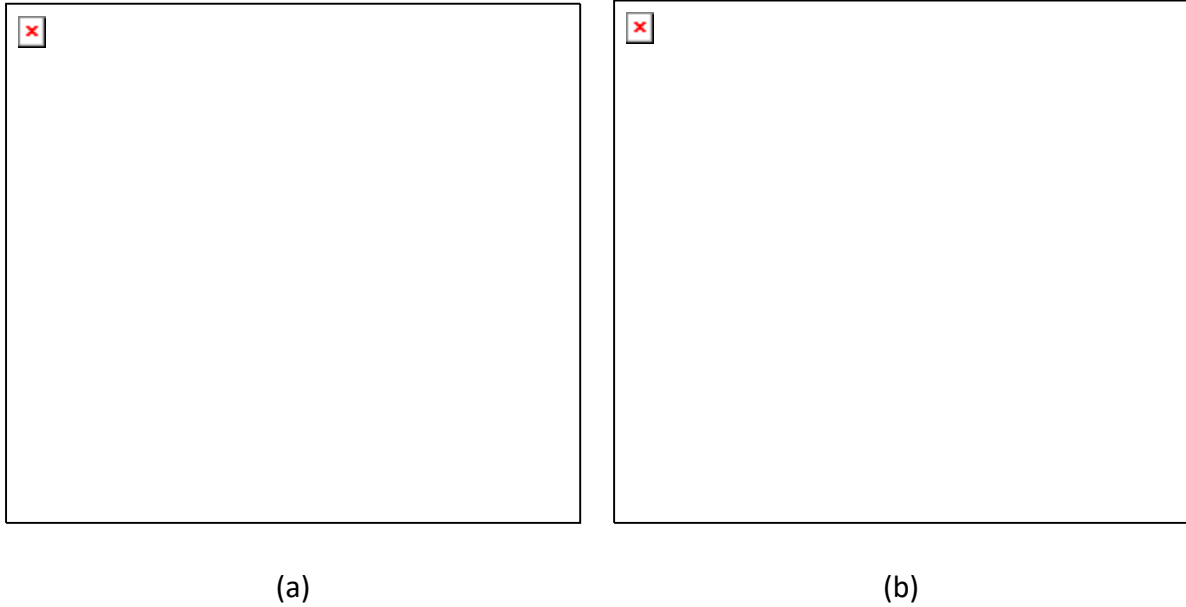


Fig. 2. Test-rig response and Bode diagram of the open loop transfer function; (a) Reference displacement input variable and (b) The friction displacement plot.

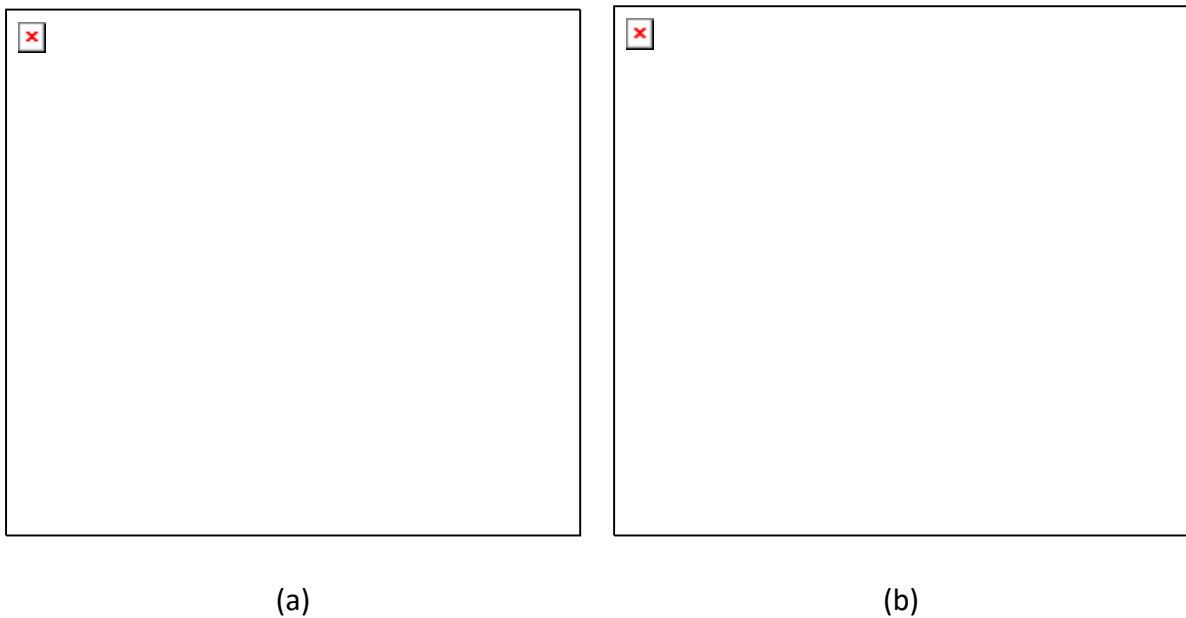


Fig. 3. Test-rig response and open loop Bode diagram of the transfer function showing too high cross-over frequency

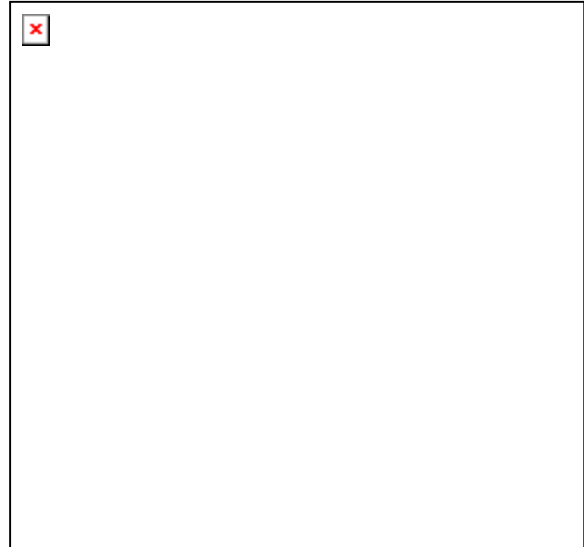


### III. Velocity Control Simulations

Velocity control simulations are further performed with the DC servo system. Thus the linear plant (DC servo) is represented by the transfer function  $G(s)$  and the linear controller described by equation 7 for the velocity control simulations. Observe that equation 7 and 6 are the same.

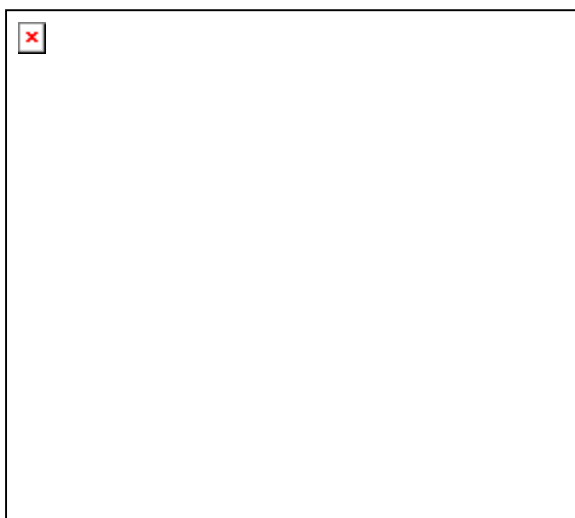


(a)



(b)

Fig. 4. (a) Response to step input with error for the new system  $K_p G(s)$  and (b) Bode plot for the new system meeting cross-over frequency requirement



(a)



(b) b



Fig. 5. Integrating the compensator network to improve system performance (a) Response meeting required steady state error and (b) Bode plot meeting required cross-over frequency

The feedback control implementation of the linear controller with the DC servo drive is performed in Matlab/Simulink software [8], [9]. The figure 6 shows the block diagram implementation of the system velocity control scheme. The transfer function of the linear controller in the block

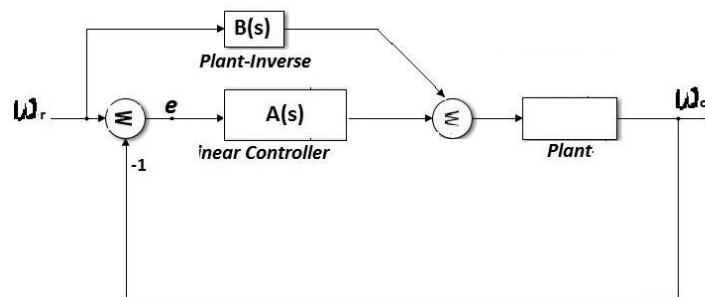


Fig. 6. Block diagram for velocity control implementation for the DC servo drive using the linear controller

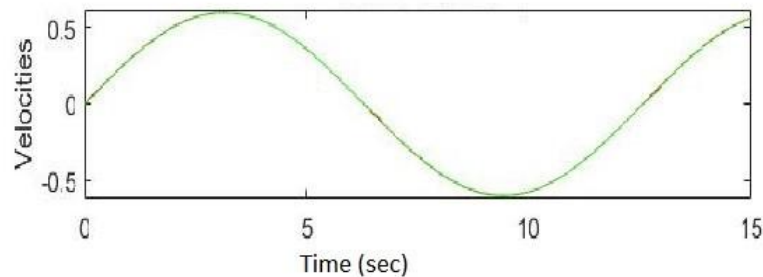


Fig. 7. Linear controller simulation result for velocity tracking of the plant showing the input (green) and output (red) velocities in (rad/sec)

diagram is given as  $A(s)$ .





$$A(s) = K_p \frac{T_i s + 1}{T_i s + \gamma} \quad (7)$$

where  $K_p = 1.5136$  is the proportional gain,  $T_i = 0.0909$  and  $\gamma = 0.0285$ .

### Velocity tracking simulation

Low amplitude, slowly varying sinusoidal velocity reference signal was used for the simulation.

The linear controller transfer function  $A(s)$  is given as in (7) and the plant inverse  $B(s) = 1/G(s)$ . With the values specified in table I, the transfer function  $G(s)$  for the DC servo becomes

$$G(s) = \frac{1.863}{0.02388s + 1} \quad (8)$$

Implementation of the linear controller without any form of friction compensation is performed and the result shown in figure 7.

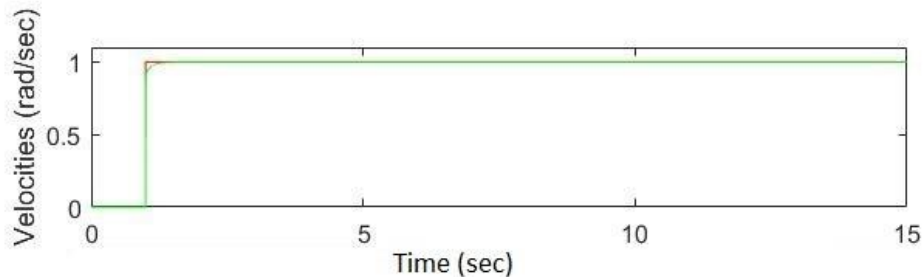


Fig. 8. Linear controller simulation result for velocity regulation of the test-bed showing the reference input (red) and output response (green) velocities in (rad/sec)

### Regulation implementation example

A step input velocity signal was used as reference with a magnitude of 1 rad/sec. The performance of the linear controller for velocity control is investigated followed by the friction observer-based controller. First the result of simulation implementation of the linear controller



for velocity control without any form of friction compensation is presented in figure 8.

#### **IV. RESULT AND DISCUSSION**

The performance of the tracking control example shown in figure 7 indicates that the linear controller was able to track variations in the reference input signal very closely and accurately. In the same vein the velocity regulation implementation result, figure 7 shows the strong correspondence between the reference input and the response signal. This is attributable to the control implementation. In this paper, the model of the DC servo drive used is linear. However, in real life the servo drive is nonlinear due to friction between the moving contacting parts. For many practical applications, the linear model could be used where very high precision velocity control is not the priority.

#### **V. CONCLUSION**

DC servo driven systems without any form of control usually require much input effort for desired output. In this paper a linear controller appropriate for velocity control purposes of a DC servo system was designed and implemented. The linear controller combines proportional with lag compensator controller. The simulation results showed the effectiveness of the linear controller for velocity tracking and regulation.

#### **REFERENCES**

- [1] Guo, R., Chen, J., & Hao, X. (2010). Position Servo Control of a DC Electromotor Using a Hybrid Method Based on Model Reference Adaptive Control (MRAC). In International Conference on Computer, Mechatronics, Control, and Electronic Engineering (CMCE) (pp. 534–537). <https://doi.org/978-1-4244-7956-6/10>
- [2] Obeidat, M. A., Wang, L. Y., & Lin, F. (2012). On-line parameter estimation of PMDC motors using binary-valued speed measurements. In 2012 IEEE Power and Energy Conference at Illinois (pp. 1–5). Ieee. <https://doi.org/10.1109/PECI.2012.6184608>
- [3] Wu, W. (2010). DC Motor Parameter Identification Using Speed Step Responses. In American Control Conference (pp. 1–5). Ieee.



- 1937–1941). Marriott Waterfront, Baltimore, MD, USA. Retrieved from  
file:///C:/Users/gf000095/Downloads/189757.pdf
- [4] Kia, M., Rezayieh, K. R., & Taherkhani, R. (2011). A novel method for measuring rotational speed of BLDC motors using voltage feedback. In The 2nd International Conference on Control, Instrumentation and Automation (pp. 791–794). Ieee. <https://doi.org/10.1109/ICCIAutom.2011.6356761>
- [5] Guarino, C., Bianco, L., & Piazzzi, A. (2002). A servo control system design using dynamic inversion. *Control Engineering Practice*, 10, 847–855. <https://doi.org/10.1.1.106.2902>
- [6] Mehmet, A. and Ismail, T. (2007). Motion Controller Design for the Speed Control of DC Servo Motor. *International Journal of Applied Mathematics and Informatics*. vol.1 issue 7.
- [7] Nagrath, I. J. and Gopal, M. (2008). *Control System Engineering*, New Age International Publishers. Fifth Edition.
- [8] Fatiha Loucif “DC motor speed control using PID controller” dept. of electrical engg and information, hunan university, Changsha, hunan, china (Email: fatiha2002@msn.com) in june 2-5 Kintex, Goggin, kora in ICCAS2005.
- [9] Aditya, P. S.(2013). Speed Control of DC Motor Using PID Controller Based on Matlab. *International Conference on Recent Trends in Applied Science with Engineering Applications*. Vol. 4, No. 6 ISSN 2222-2871