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TRANSIENT ANALYSIS OF TWO-DIMENSIONAL M/M/1 QUEUEING MODEL WITH REPAIRABLE SERVER AND BERNOULLI SCHEDULE

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Abstract: *The present work concerned with the transient analysis of a **two-dimensional state** markovian queueing model in which, when a customer has just been served and other customers are present, the server accepts a customer with fix probability p or commences a vacation of random duration with probability (1- p). Whenever no customers are present, after a service completion or a vacation completion, the server always takes a vacation with probability one. And also the server is subject to random breakdowns while in operation and must be repaired before service can resume. The servers' repair times, the servers' service times, vacation times, and breakdown times are exponentially distributed. Arriving units are in Poisson stream. And it is shown that the transient state probabilities can be easily computed with recurrence relations. In order to validate the analytical approach, we compute numerical results. Graphical representation is also performed to explore the effect of different parameters.*

Keywords: *Two-dimensional queueing model; Bernoulli schedule; Multiple vacation; Beakdown; Repair; Laplace transform.*

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1. INTRODUCTION

The most classical case in a queue assumes a reliable machine or server; however, in practice we often meet the cases where the servers may fail and can be repaired. Besides, server breakdowns are considered as the most natural cause of service interruptions. Server breakdowns phenomena can be encountered in the area of computer, communication network and flexible manufacturing systems, etc. The performance of a system may be heavily affected by the service station breakdowns and limited repair capacity; as such queueing systems with a repairable server are of worth investigation from the performance prediction viewpoint. The problem of queueing systems with server failures is of continuing interest to many researchers. Several models have been built and analyzed. White and Christie **[1]** introduced queueing systems with service interruptions. Further these results are extended by many researchers like Aissani and Artalejo**[2]**, Avi-Itzhak et al.**[3]**, Gaver**[4]**, Thirvengadam**[5]**, Takine and Sengupta**[6]**, Federgruen and Green**[7]** and Mitrany and Avi-Itzhak**[8]**. Also, Wang **[9, 10, 11]** extended the markovian queueing model under the N policy with server breakdowns and summarized the major developments in this area. Krishna Kumar et al. **[12]** also considered this variant queueing model with an unreliable server.

Also, in many queueing models the server becomes unavailable for occasional intervals of time. In literature, a time interval when the server is either unavailable or idle is called vacation period. Queueing model with server vacations have attracted much attention from numerous researchers since Levy and Yechiali **[13]**. Server vacations are useful for the systems in which the server wants to utilize his idle time for different purposes. An excellent survey of queueing systems with server vacations was found in Doshi **[14]**. Cooper **[15]**, Takagi **[16]** and Tian and Zhang **[17]** also presented various vacation models. Further, various authors studied queues with server vacations under various vacation policies including Bernoulli schedules. The classical vacation scheme with Bernoulli schedule in which the server serves the new customer with probability p or takes a vacation with probability (1-p) was originated and developed significantly by Keilson and Servi **[18]**. The advantage of the Bernoulli schedule is the existence of a control parameter p. By adjusting the value of p, we can control the congestion of the system. Various aspects of Bernoulli vacation models for single server queueing systems have also been studied by Servi **[19]** and Ramaswamy and Servi **[20]**.

The queueing models under random breakdowns have also been studied by including server vacations. Queueing models with server breakdowns and vacations accommodate the real-world situations more closely. Grey et al. **[21]** considered a vacation queueing model with service breakdowns. In this respect, Ke **[22, 23]** has done good deal of work on batch arrival queue with server breakdown and multiple vacations. All of the above mentioned contributions are only confined to results describing steady-state operation and rely on different assumptions for the queueing models. But, in this paper we put emphasis on transient analysis because steady state measures do not reveal the complete picture of the system behaviour.

In many potential applications of queueing theory, the practitioner needs to know how the system will operate up to some time instant t. Further, if the system is empty initially, the fraction of time the server is busy and the initial rate of output etc., will be below the steady state values and hence the use of steady state results to obtain these measures is not appropriate. Thus, the investigation of the transient behaviour of the queueing model is also important from the point of view of theory as well as applications. Krishna Kumar et al. **[24]** investigated the transient behaviour of M/M/1 queueing model with catastrophes & breakdowns.

In the present work, we consider a single-server **two-dimensional** markovian queue with Repairable server and Bernoulli Schedule for the probability that exactly i arrivals and j services occur over a time interval of length t in a queueing model that the server is on vacation at the beginning of the interval, in order to obtain some analytical results that do not appear to be present in the literature. The principal purpose of our work is to realize an extensive analysis of the system from both queueing and reliability points of view. Since many applications of queueing theory involve queues which are emptied and restarted periodically and thus not susceptible to analysis using the well-known equilibrium results, there are many potential applications for results obtained.

The rest of this paper is organized as follows. Section 2 gives a relatively formal description of the queueing model. In Section 3, we define the two-dimensional state model and derive the difference-differential equations and time dependent solution is also obtained for our model. Some special cases are discussed in Section 4. Section 5 presents the performances measures with numerical results where we provide a variety of tables for different values of the model parameters and also we have numerically verified our results in some special cases that exists in the literature. Some graphs are presented showing the effect of model parameters on some performance measures in Section 6.

2. MODEL DESCRIPTION

2.1 Assumptions and Notations:

- The arrivals follow a Poisson distribution with parameter λ.
- The service times are exponentially distributed with parameter μ.
- The vacation time, mean life time and mean repair time of the service channel follow an exponential distribution with parameters w, α and β respectively.
- Various stochastic processes involved in the system are statiscally independent.
- Service discipline is First Come First Served (FCFS).

Initially, there are no units in the system and the server is on vacation, i.e.

$$P_{0,0,V}(0)=1\ ;\ P_{0,0,B}(0)=0\ ;\ P_{0,0,R}(0)=0 \qquad \textbf{(2.1)}$$

$$\delta_{i,j}=\begin{cases}1; & \text{when} \quad i=j\\ 0; & \text{when} \quad i\neq j\end{cases}$$

Laplace transform of F (t) is

$$\overline{F}(s)=\int_0^\infty e^{-st}\,F(t)\,dt \qquad ;\ \mathrm{Re}(s)>0 \qquad \textbf{(2.2)}$$

$$\overline{N}^{a,b}_{n_1,n_2}(s)=\frac{1}{(s+a)^{n_1}(s+b)^{n_2}} \qquad \textbf{(2.3)}$$

$$\overline{G}^{a,b,c,d}_{n_1,n_2,n_3,n_4}(s)=\frac{1}{(s+a)^{n_1}(s+b)^{n_2}(s+c)^{n_3}(s+d)^{n_4}} \qquad \textbf{(2.4)}$$

The Laplace inverse of

$$\frac{Q(p)}{P(p)}\ \text{is}\ \sum_{k=1}^{n}\sum_{\ell=1}^{m_k}\frac{t^{m_k-\ell}e^{a_k t}}{(m_k-\ell)!(\ell-1)!}\times\frac{d^{\ell-1}}{dp^{\ell-1}}\frac{Q(p)}{P(p)}(p-a_k)^{m_k}\Bigg|_{p=a_k} \quad a_i\neq a_k\ \ \text{for}\ i\neq k$$

Where, $P(p)=(p-a_1)^{m1}(p-a_2)^{m2}\ldots(p-a_n)^{mn}$

$Q(p)$ is polynomial of degree $<m_1+m_2+m_3+\ldots+m_n-1$

$$N^{a,b}_{n_1,n_2}(t)=\sum_{m=1}^{n_2}\frac{t^{n_2-m}e^{-at}(-1)^{m+1}}{(n_2-m)!(m-1)!(b-a)^{n_1+m-1}}\left(\prod_{g=0}^{m-2}(n_1+g)\right)^{1-\delta_{m,1}}+\sum_{m=1}^{n_1}\frac{t^{n_1-m}e^{-bt}(-1)^{m+1}}{(n_1-m)!(m-1)!(a-b)^{n_2+m-1}}\left(\prod_{g=0}^{m-2}(n_2+g)\right)^{1-\delta_{m,1}} \quad (2.5)$$

$$\begin{aligned}G^{a,b,c,d}_{n_1,n_2,n_3,n_4}(t)=&\sum_{\ell=1}^{n_1}\sum_{m=1}^{\ell}\sum_{n=1}^{m}\frac{e^{-at}t^{n_1-\ell}(-1)^{\ell+1}}{(n_1-\ell)!(\ell-m)!}\frac{\left(\prod_{g_1=0}^{\ell-m-n}(n_4+g_1)\right)^{1-\delta_{\ell,m}}\left(\prod_{g_2=0}^{m-n-1}(n_3+g_2)\right)^{1-\delta_{m-n,0}}\left(\prod_{g_3=0}^{n-2}(n_2+g_3)\right)^{1-\delta_{n,1}}}{(n-1)!(m-n)!(b-a)^{n_2+n-1}(c-a)^{n_3+m-n}(d-a)^{n_4+\ell-m}}\\
&+\sum_{\ell=1}^{n_2}\sum_{m=1}^{\ell}\sum_{n=1}^{m}\frac{e^{-bt}t^{n_2-\ell}(-1)^{\ell+1}}{(n_2-\ell)!(\ell-m)!}\frac{\left(\prod_{g_1=0}^{\ell-m-n}(n_4+g_1)\right)^{1-\delta_{\ell,m}}\left(\prod_{g_2=0}^{m-n-1}(n_3+g_2)\right)^{1-\delta_{m-n,0}}\left(\prod_{g_3=0}^{n-2}(n_1+g_3)\right)^{1-\delta_{n,1}}}{(n-1)!(m-n)!(a-b)^{n_1+n-1}(c-b)^{n_3+m-n}(d-b)^{n_4+\ell-m}}\\
&+\sum_{\ell=1}^{n_3}\sum_{m=1}^{\ell}\sum_{n=1}^{m}\frac{e^{-ct}t^{n_3-\ell}(-1)^{\ell+1}}{(n_3-\ell)!(\ell-m)!}\frac{\left(\prod_{g_1=0}^{\ell-m-n}(n_4+g_1)\right)^{1-\delta_{\ell,m}}\left(\prod_{g_2=0}^{m-n-1}(n_2+g_2)\right)^{1-\delta_{m-n,0}}\left(\prod_{g_3=0}^{n-2}(n_1+g_3)\right)^{1-\delta_{n,1}}}{(n-1)!(m-n)!(a-c)^{n_1+n-1}(b-c)^{n_2+m-n}(d-c)^{n_4+\ell-m}}\\
&+\sum_{\ell=1}^{n_4}\sum_{m=1}^{\ell}\sum_{n=1}^{m}\frac{e^{-dt}t^{n_4-\ell}(-1)^{\ell+1}}{(n_4-\ell)!(\ell-m)!}\frac{\left(\prod_{g_1=0}^{\ell-m-n}(n_3+g_1)\right)^{1-\delta_{\ell,m}}\left(\prod_{g_2=0}^{m-n-1}(n_2+g_2)\right)^{1-\delta_{m-n,0}}\left(\prod_{g_3=0}^{n-2}(n_1+g_3)\right)^{1-\delta_{n,1}}}{(n-1)!(m-n)!(a-d)^{n_1+n-1}(b-d)^{n_2+m-n}(c-d)^{n_3+\ell-m}}\end{aligned} \quad (2.6)$$

3. THE TWO-DIMENSIONAL STATE MODEL

Nomenclature

$P_{i,j,V}(t)$ = The probability that there are exactly i arrivals and j departures by time t and the server is on vacation, $j \le i$

$P_{i,j,B}(t)$ = The probability that there are exactly i arrivals and j departures by time t and the server is busy in relation to the queue, $j<i$

$P_{i,j,R}(t)$ = The probability that there are exactly i arrivals and j departures by time t and the server is broken down and is under repair, $j<i$

$P_{i,j}(t)$ = The probability that there are exactly i arrivals and j departures by time t, $j \le i$

3.1 The difference-differential equations governing the system are:

$$\frac{d}{dt}P_{i,i,V}(t)=-\lambda P_{i,i,V}(t)+\mu P_{i,i-1,B}(t)(1-\delta_{i,0}) \quad ; i\ge 0 \quad (3.1)$$

$$\frac{d}{dt}P_{i,j,V}(t)=-(\lambda+w)P_{i,j,V}(t)+\lambda P_{i-1,j,V}(t)+\mu(1\text{-}p)P_{i,j-1,B}(t)(1-\delta_{j,0}) \quad ; i>j\ge 0 \quad (3.2)$$

$$\frac{d}{dt}P_{i,j,B}(t)=-(\lambda+\mu+\alpha)P_{i,j,B}(t)+\lambda P_{i-1,j,B}(t)(1\text{-}\delta_{i-1,j})+\mu p P_{i,j-1,B}(t)(1-\delta_{j,0})+\beta P_{i,j,R}(t)+wP_{i,j,V}(t) \quad ; i>j\ge 0 \quad (3.3)$$

$$\frac{d}{dt}P_{i,j,R}(t) = -(\lambda+\beta)\,P_{i,j,R}(t) + \lambda\,P_{i-1,j,R}(t)\,(1-\delta_{i-1,j}) + \alpha\,P_{i,j,B}(t) \qquad ;i>j\geq 0 \quad (3.4)$$

Clearly,

$$P_{i,j}(t) = P_{i,j,V}(t) + P_{i,j,B}(t)(1-\delta_{i,j}) + P_{i,j,R}(t)(1-\delta_{i,j}) \qquad ;i\geq j\geq 0 \quad (3.5)$$

Taking the Laplace transform of equations (3.1) to (3.4) along with (2.1) and solving recursively, we have,

$$\overline{P}_{0,0,V}(s) = \frac{1}{s+\lambda} \qquad (3.6)$$

$$\overline{P}_{i,0,V}(s) = \left(\frac{\lambda}{s+\lambda+w}\right)^{i}\left(\frac{1}{s+\lambda}\right) \qquad ;\ i>0 \quad (3.7)$$

$$\overline{P}_{i,0,B}(s) = \sum_{m_0=0}^{i-1}\ \sum_{m_1=0}^{i-m_o-2+(\delta_{m_0,i-1})}\ \sum_{m_2=1-\delta_{m_1,0}}^{i-m_1-m_0-1}\ \sum_{n=1}^{\infty}\lambda^{i}w(\alpha\beta)^{n+m_2-1}\binom{(i-m_0-m_1-1)}{m_2}\frac{1}{m_1!}\left(\prod_{r=0}^{m_1-1}(m_2+r)\right)^{(1-\delta_{m_1,0})}$$
$$\left(\prod_{g=0}^{i-m_0-m_1-2}(n+g)\right)^{(1-\delta_{i-m_0-m_1,1})}\ \overline{G}^{\lambda+w,\ \lambda+\beta,\ \lambda+\mu+\alpha,\ \lambda}_{m_0+1,\ m_1+2m_2+n-1,\ n-1,\ 1}(s) \qquad ;i>0 \quad (3.8)$$

$$\overline{P}_{i,0,R}(s) = \sum_{m_0=0}^{i-1}\ \sum_{m_1=0}^{i-m_0-1}\ \sum_{m_2=1}^{i-m_0-m_1}\ \sum_{n=1}^{\infty}\lambda^{i}w\alpha^{n+m_2}\beta^{n+m_2-1}\frac{1}{m_1!}\binom{(i-m_0-m_1-1)}{(m_2-1)}$$
$$\left(\prod_{r=0}^{m_1-1}(m_2+r)\right)^{(1-\delta_{m_1,0})}\left(\prod_{g=0}^{i-m_0-m_1-2}(n+g)\right)^{(1-\delta_{i-m_0-m_1,1})}\ \overline{G}^{\lambda+w,\ \lambda+\beta,\ \lambda+\mu+\alpha,\ \lambda}_{m_0+1,\ m_1+2m_2+n-3,\ n-1,\ 1}(s) \qquad ;\ i>0 \quad (3.9)$$

$$\overline{P}_{i,j,V}(s) = \sum_{m=0}^{i-j}\lambda^{i-j-m}m\,(1-p)^{1-\delta_{m,0}}\,\overline{N}^{\lambda+w,\ \lambda}_{i-j+1-m-\delta_{m,0},\ \delta_{m,0}}(s)\,\overline{P}_{j+m,j-1,B}(s) \qquad ;\ i\geq j>0 \quad (3.10)$$

$$\overline{P}_{i,j,R}(s) = \sum_{m=1}^{i-j}\lambda^{i-j-m}\alpha\left(\frac{1}{s+\lambda+\beta}\right)^{i-j-k+1}\overline{P}_{j+m,\ j,\ B}(s) \qquad ;\ i>j\geq 0 \quad (3.11)$$

$$\overline{P}_{i,\ j,\ B}(s) = \sum_{k=1}^{i-j}\lambda^{i-j-k}\sum_{n=0}^{\infty}(\alpha\beta)^{n}\left[\begin{array}{l}\mu\,p\,\overline{N}^{n,\ n+1}_{\lambda+\beta,\ \lambda+\mu+\alpha}(s)\,\overline{P}_{j+k,\ j-1,\ B}(s) + \mu w\sum_{m=0}^{k}\lambda^{k-m}(1-p)^{1-\delta_{m,0}}\\ \overline{G}^{n,\ n+1,\ (k-m+1-\delta_{m,0}),\ (\delta_{m,0})}_{\lambda+\beta,\ \lambda+\mu+\alpha,\ \lambda+w,\ \lambda}(s)\,\overline{P}_{j+m,\ j-1,\ B}(s)\end{array}\right] \qquad ;i>j\geq 0 \quad (3.12)$$

Taking the Laplace Inverse transform of equations (3.6) to (3.12), we have

$$P_{0,0,V}(t) = e^{-\lambda t} \quad \textbf{(3.13)}$$

$$P_{i,0,V}(t)=\lambda^i e^{-\lambda t}\left[\frac{1}{w^i}-e^{-wt}\sum_{k=0}^{i-1}\frac{t^k}{k!}\frac{1}{w^{i-k}}\right] \quad \textbf{; i >0 (3.14)}$$

$$P_{i,0,B}(t) = \sum_{m_0=0}^{i-1}\ \sum_{m_1=0}^{i-m_0-2+(\delta_{m_0,i-1})}\ \sum_{m_2=1-\delta_{m_1,0}}^{i-m_1-m_0-1}\ \sum_{n=1}^{\infty}\lambda^i w(\alpha\beta)^{n+m_2-1}\frac{1}{m_1!}\binom{(i-m_0-m_1-1)}{m_2}$$

$$\left(\prod_{r=0}^{m_1-1}(m_2+r)\right)^{(1-\delta_{m_1,0})}\left(\prod_{g=0}^{i-m_0-m_1-2}(n+g)\right)^{(1-\delta_{i-m_0-m_1,1})} G^{\lambda+w,\ \lambda+\beta,\ \lambda+\mu+\alpha,\lambda}_{m_0+1,\ m_1+2m_2+n-1,\ n-1,1}(t) \quad \textbf{; i>0 (3.15)}$$

$$P_{i,0,R}(t) = \sum_{m_0=0}^{i-1}\ \sum_{m_1=0}^{i-m_0-1}\ \sum_{m_2=1}^{i-m_0-m_1}\ \sum_{n=1}^{\infty}\lambda^i\ w\ \alpha^{n+m_2}\ \beta^{n+m_2-1}\frac{1}{m_1!}\binom{(i-m_0-m_1-1)}{(m_2-1)}$$

$$\left(\prod_{r=0}^{m_1-1}(m_2+r)\right)^{(1-\delta_{m_1,0})}\left(\prod_{g=0}^{i-m_0-m_1-2}(n+g)\right)^{(1-\delta_{i-m_0-m_1,1})} G^{\lambda+w,\ \lambda+\beta,\ \lambda+\mu+\alpha,\ \lambda}_{m_0+1,\ m_1+2m_2+n-3,\ n-1,\ 1}(t) \quad \textbf{; i>0 (3.16)}$$

$$P_{i,j,R}(t) = \sum_{m=1}^{i-j}\left(\frac{\alpha(\lambda t)^{i-j-m}}{(i-j-m)!}\right)e^{-(\lambda+\beta)t} * P_{j+m,j,B}(t) \quad \textbf{; i>j ≥ 0 (3.17)}$$

$$P_{i,j,V}(t)=\sum_{m=0}^{i-j}\lambda^{i-j-m}\mu\,(1-p)^{1-\delta_{m,0}}N^{\lambda+w,\ \lambda}_{i-j+1-m-\delta_{m,0},\ \delta_{m,0}}(t)*P_{j+m,j-1,B}(t) \quad \textbf{; i ≥ j>0 (3.18)}$$

$$P_{i,j,B}(t)=\sum_{k=1}^{i-j}\lambda^{i-j-k}\sum_{n=0}^{\infty}(\alpha\beta)^n\left[\begin{array}{l}\mu\, p\, N^{n,\ n+1}_{\lambda+\beta,\ \lambda+\mu+\alpha}(t)*P_{j+k,\ j-1,\ B}(t)+\mu w\sum_{m=0}^{k}\lambda^{k-m}(1-p)^{1-\delta_{m,0}}\\ G^{n,\ n+1,\ (k-m+1-\delta_{m,0}),\ (\delta_{m,0})}_{\lambda+\beta,\ \lambda+\mu+\alpha,\ \lambda+w,\ \lambda}(t)*P_{j+m,\ j-1,\ B}(t)\end{array}\right] \quad \textbf{; i>j ≥ 0 (3.19)}$$

4. SPECIAL CASES

4.1 When server takes the vacation only, i.e. by letting $\alpha \to 0$ and $\beta \to \infty$, obtained results agree with the results of Indra & Renu **[25]**.

4.2 Along with the case 4.1, p=1 in eqns. (3.1) to (3.4), then above described model reduces to exhaustive service discipline and obtained results coincide with results on Indra **[26]**.

4.3 Along with the case-4.2, when the server is instantaneously available i.e. no discipline of vacation. Letting $w \to \infty$ in eqns. (3.1) to (3.4), we have

$$P_{0,0}(t) = P_{0,0,V}(t) = e^{-\lambda t}$$

$$P_{i,j}(t) = P_{i,j,B}(t)=\left(\frac{\lambda}{\mu}\right)^i\frac{(\mu t)^j e^{-\lambda t}}{i!}\sum_{K=0}^{j}\frac{(i-k)}{k!}\sum_{m=0}^{j-k}\left(\frac{(-1)^m(m+i+k)!}{m!(j-k-m)!(\mu t)^{m+k}}\right)\left(1-e^{-\mu t}\sum_{r=0}^{m+i+k-1}\frac{(\mu t)^r}{r!}\right)$$

$;i > j \geq 0$

Then results coincide with eqn. (10) of Pegden and Rosenshine **[27]**.

5. PERFORMANCES MEASURES OF THE SYSTEM

5.1 The Laplace transform $\overline{P}_{i\bullet}(s)$ of the probability $P_{i\bullet}(t)$ that exactly i units arrive by time t is;

$$\overline{P}_{i\bullet}(s) = \sum_{j=0}^{i}\left\{\overline{P}_{i,j,V}(s) + \overline{P}_{i,j,B}(s)(1-\delta_{i,j}) + \overline{P}_{i,j,R}(s)(1-\delta_{i,j})\right\} = \sum_{j=0}^{i}\overline{P}_{i,j}(s) = \frac{\lambda^{i}}{(s+\lambda)^{i+1}} \quad ;i > 0 \quad (5.1)$$

And its Inverse Laplace transform is $P_{i\bullet}(t) = \sum_{j=0}^{i} P_{i,j}(t) = \frac{(\lambda t)^{i} e^{-\lambda t}}{i!}$ **(5.2)**

The Laplace transform of the mean number of the arrivals is $\sum_{i=0}^{\infty} i\overline{P}_{i,\bullet}(s) = \left(\frac{\lambda}{s^{2}}\right)$ **(5.3)**

The arrivals follow a Poisson distribution as the probability of the total number of arrivals is not affected by the vacation times and breakdowns of the server.

$$\sum_{i=0}^{\infty}\sum_{j=0}^{i}\left\{\overline{P}_{i,j,V}(s) + \overline{P}_{i,j,B}(s)(1-\delta_{i,j}) + \overline{P}_{i,j,R}(s)(1-\delta_{i,j})\right\} = \frac{1}{s} \quad (5.4)$$

$$\sum_{i=0}^{\infty}\sum_{j=0}^{i}\left\{P_{i,j,V}(t) + P_{i,j,B}(t)(1-\delta_{i,j}) + P_{i,j,R}(t)(1-\delta_{i,j})\right\} = 1 \quad (5.5)$$

Hence, a verification

And the numerical results for the probabilities of exact number of arrivals when the server is busy i.e. $\sum_{j=0}^{i} P_{i,j,B}(t)$, when the server is on vacation i.e. $\sum_{j=0}^{i} P_{i,j,V}(t)$, when the server is under repair i.e. $\sum_{j=0}^{i} P_{i,j,R}(t)$, are computed for different sets of parameter and are summarized in Table–1. Table-1 is based on the relationship (5.1) and its last column shows complete agreement with the Table–1 of Pegden and Rosenshine [27].

Table 1; Probability that exactly i units have arrived by time t (α=1; β=1.5; w=0.8; p=0.4, t=3)

λ	μ	i	$\frac{e^{-\lambda t}(\lambda t)^i}{i!}$	$\sum_{j=0}^{i} P_{i,j,V}(t)$	$\sum_{j=0}^{i} P_{i,j,B}(t)$	$\sum_{j=0}^{i} P_{i,j,R}(t)$	$\sum_{j=0}^{i} P_{i,j}(t)$
1	2	1	0.1493612	0.119434486	0.01926033488	0.01066638381	0.14936120510
1	2	3	0.2240418	0.131916122	0.05864113138	0.03348455393	0.22404180761
1	2	5	0.0952209	0.013574327	0.04022048963	0.04142616719	0.09522098419
2	3	1	0.0148725	0.012728398	0.00136811683	0.00077599756	0.01487251306
2	3	3	0.0892350	0.060918281	0.01796349851	0.01035329816	0.08923507835
2	3	5	0.1606231	0.099333419	0.03835008745	0.02293963459	0.16062314105
3	4	1	0.0011106	0.000986361	0.00007893639	0.00004539077	0.00111068823
3	4	3	0.0149942	0.011154984	0.00242938643	0.00140992071	0.01499429119
3	4	5	0.0607268	0.041514843	0.01201341402	0.00719862206	0.06072687934
4	5	1	0.0000737	0.000067000	0.00000425946	0.00000247048	0.00007373054
4	5	3	0.0017695	0.001391323	0.00023887921	0.00013933085	0.00176953315
4	5	5	0.0127406	0.009311297	0.00214333146	0.00128600978	0.01274063873

5.2 The numerical results for the probabilities that exactly j number of customers have been served when the server is on vacation i.e. $\sum_{i=j}^{\infty} P_{i,j,V}(t)$, when the server is busy i.e. $\sum_{i=j}^{\infty} P_{i,j,B}(t)$ are computed for different sets of parameters ($\lambda = 2, \mu = 3$, w=2, t=2, p=0.4, 0.6, 0.8) and are based on the relationship $P_{\bullet j}(t)=\sum_{i=j}^{\infty} P_{i,j}(t)$ where $P_{i,j}(t)$ is defined in equation (3.5). By adjusting the value of p, we can control the congestion of the system. And from the numerical results it is obvious that as p increases the probability of departures increases when the server is busy. In figs.5.1-5.2, the graphical representation of $P_{\bullet j}(t)$ with the variation of p has been shown.

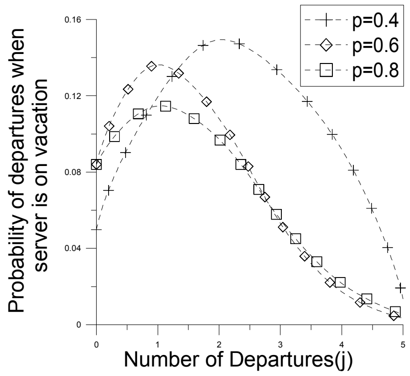

fig.5.1

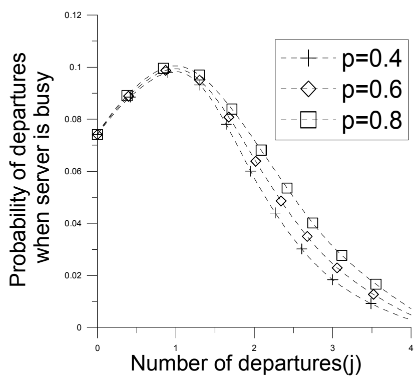

fig. 5.2

5.3 The probability of exactly n customers in the system at time t, denoted by $P_n(t)$ can be expressed in terms of $P_{i,j}(t)$. And are based on the relationship $P_n(t)=\sum_{j=0}^{\infty} P_{j+n,j}(t)$ where

$P_n(t)=P_{n,V}(t)+P_{n,B}(t)+P_{n,R}(t)$ and $P_{n,V}(t)$

(i) Customers when the server is busy, i.e. $P_{n,B}(t)=\sum_{j=0}^{\infty} P_{j+n,j,B}(t)$

(ii) Customers when the server is on vacation, i.e $P_{n,V}(t)=\sum_{j=0}^{\infty} P_{j+n,j,V}(t)$

(iii) Customers when the server is under repair, i.e $P_{n,R}(t)=\sum_{j=0}^{\infty} P_{j+n,j,R}(t)$.

$P_{n,V}(t)$, $P_{n,B}(t)$, $P_{n,R}(t)$ and $P_n(t)$ are computed for different values of parameters ($\lambda=2, \mu_V=2, \mu_B=3$, w=2, p=0.4). In figs. 5.3 to 5.6, the graphical representation of $P_{n,V}(t)$, $P_{n,B}(t)$, $P_{n,R}(t)$ and $P_n(t)$ with the variation of time t has been shown.

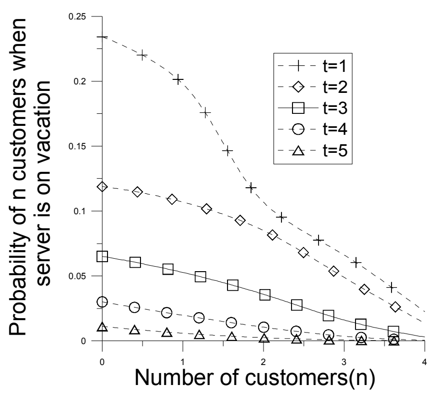

fig. 5.3

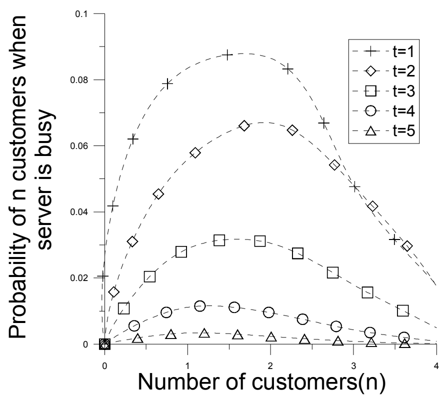

fig. 5.4

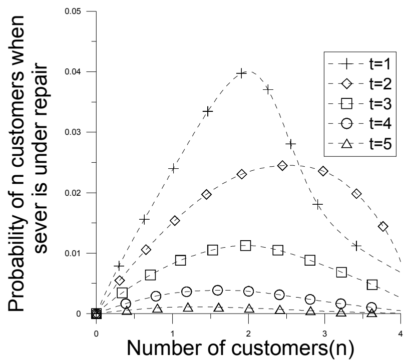

fig. 5.5

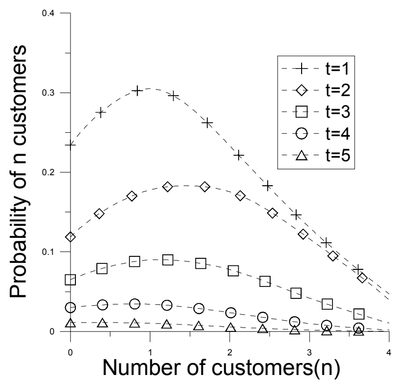

fig. 5.6

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5.4 The server's utilization time, server's breakdown time and the server's vacation time i.e. the fraction of time the server is busy & the fraction of time the server is broken down & the fraction of time the server is on vacation until time t can also be expressed in terms of $P_{i,j}(t)$.

Thus the server's utilization time is U(t)= $\sum_{i=0}^{\infty}\sum_{j=0}^{i}P_{i,j,B}(t)$. And the server's breakdown's time in B(t)= $\sum_{i=0}^{\infty}\sum_{j=0}^{i}P_{i,j,R}(t)$. And the server's vacation time is V(t)= $\sum_{i=0}^{\infty}\sum_{j=0}^{i}P_{i,j,V}(t)$.

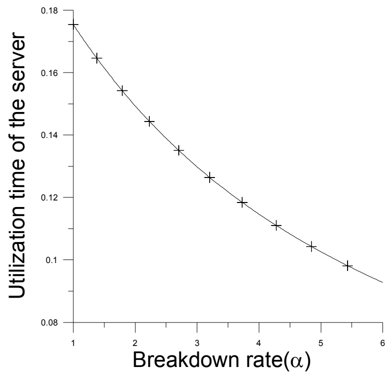

fig. 5.7

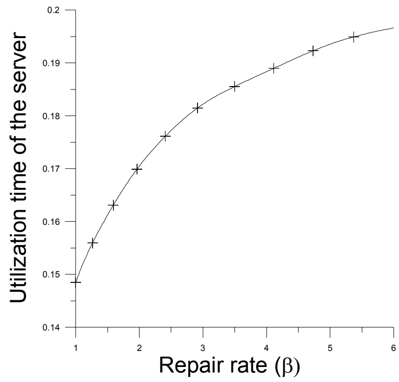

fig. 5.8

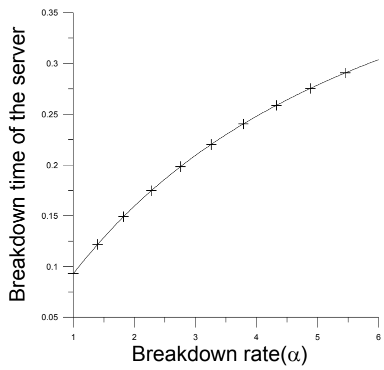

fig. 5.9

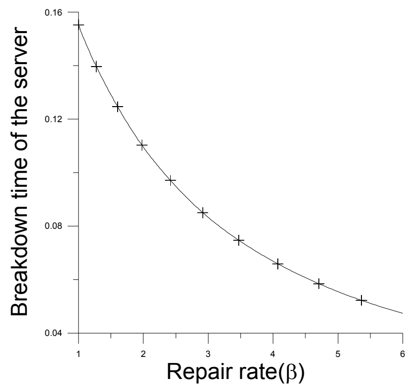

fig. 5.10

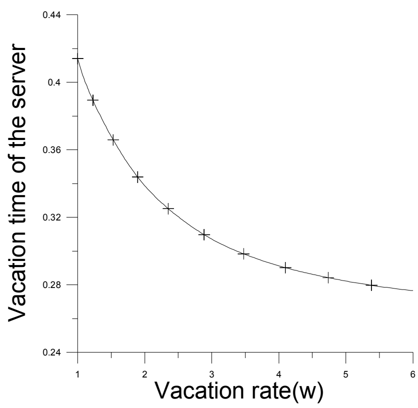

fig. 5.11

In figures 5.7-5.10, Utilization time and Breakdown time decreases with the increase in α and β and increases with the increase in β and α. And in fig. 5.11 as w increases Vacation time of the server decreases i.e. the server is instantly available.

CONCLUSION

Repairable server with Bernoulli Schedule is often used for the performance prediction of many real time systems. We have examined the effect of various parameters namely the probability p, time, failure rate, repair rate and vacation rate, etc. by taking numerical illustration. The system performance measures supply better insight into the behavior of a queueing system than the probability of exact number of units in the system at a given time, studied in early literature on queues, in many practical situations and is therefore more justified.

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