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Numerical Simulation of Journal Bearings Lubricated with Couple Stress and Micropolar Fluids

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Abstract

This study presents a comprehensive numerical analysis of journal bearings lubricated with couple stress and micropolar fluids. Modified Reynolds equations were derived for both fluid models to account for the non-Newtonian behaviour of the lubricants, and the finite difference method (FDM) was employed to solve these equations. The analysis also includes the energy equation to account for thermal effects due to viscous dissipation. The results reveal that micropolar fluids provide a higher load-carrying capacity, while couple stress fluids exhibit smoother pressure distribution and lower temperature rise, making them ideal for applications where uniform lubrication and temperature control are critical. The performance of both fluids was evaluated, offering insights into the optimal use of non-Newtonian lubricants in high-performance bearing applications.

Keywords: Couple stress fluids, Micropolar fluids, Journal bearings, Hydrodynamic lubrication, Modified Reynolds equation, Viscous dissipation, Load-carrying capacity, Temperature rise, Non-Newtonian fluids, Numerical simulation, Finite difference method (FDM).

1. Introduction

1.1. Background and Motivation

Journal bearings are widely used in rotating machinery, supporting radial loads and enabling smooth rotational motion between a shaft and a stationary component. The performance of journal bearings depends on the formation of a thin film of lubricant that prevents metal-to-metal contact, reducing friction and wear. Traditional lubrication theory often relies on Newtonian fluids, where the viscosity remains constant regardless of the shear rate. However, in high-performance applications, Newtonian fluids may not provide sufficient lubrication, especially under extreme pressure and temperature conditions (Pinkus & Sternlicht, 1961).

Couple stress and micropolar fluids are examples of non-Newtonian fluids that provide improved performance in journal bearings. Couple stress fluids introduce a characteristic length scale to account for the internal microstructure of the lubricant, which enhances the load-carrying capacity and reduces friction (Stokes, 1966). Micropolar fluids, an extension of couple stress fluids, account for both microstructural effects and the microrotation of fluid particles, providing an even more detailed description of fluid behavior (Eringen, 1966). This study aims to analyze the hydrodynamic performance of journal bearings lubricated with these two non-Newtonian fluids using numerical simulation techniques.

1.2. Objectives of the Study

The primary objectives of this study are:

1. To derive the modified Reynolds equations for journal bearings lubricated with couple stress and micropolar fluids.

2. To numerically solve these equations using the finite difference method (FDM) and compare the performance of both fluids in terms of pressure distribution, load-carrying capacity, and temperature rise.

3. To analyze the impact of temperature-dependent viscosity and cavitation on the lubrication performance.

2. Mathematical Modeling of Lubrication with Couple Stress and Micropolar Fluids

The lubrication performance of journal bearings can be modeled using the Reynolds equation, which describes the pressure distribution in the lubricant film. For non-Newtonian fluids, such as couple stress and micropolar fluids, the classical Reynolds equation is modified to incorporate the effects of fluid microstructure and microrotations.

2.1. Modified Reynolds Equation for Couple Stress Fluids

Couple stress fluids introduce an additional length scale, l_c, which accounts for the interactions between fluid particles at the microscale. The governing equation for the flow of couple stress fluids in journal bearings is derived from the Navier-Stokes equations with additional terms for couple stresses. The Cauchy stress tensor σ_{ij} for couple stress fluids is given by (Stokes, 1966):

$$\sigma_{ij} = -p\delta_{ij} + 2\mu \frac{\partial v_i}{\partial x_j} + \eta \left(\frac{\partial^2 v_i}{\partial x_j^2} \right)$$

Where:

- p is the pressure,
- μ is the dynamic viscosity of the fluid,
- v_i is the velocity component in the i-direction,
- η is the couple stress coefficient, related to the characteristic length scale l_c.

The film thickness $h(\theta)$ in a journal bearing is given by:

$$h(\theta) = c + e\cos(\theta)$$

Where:

- c is the radial clearance between the journal and bearing,
- e is the eccentricity, representing the displacement of the journal center from the bearing center,
- θ is the angular position around the journal.

The modified Reynolds equation for couple stress fluids is:

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y} \right) + \frac{12\mu U h}{h^2 + 12 l_c^2} = 0$$

Where:

- $p(x, y)$ is the pressure distribution,
- U is the sliding velocity between the journal and the bearing surface,
- l_c is the characteristic length scale of the couple stress fluid.

2.2. Modified Reynolds Equation for Micropolar Fluids

Micropolar fluids account for both translational and rotational motions of fluid particles, with the microrotation represented by the vector ω. The governing equations for micropolar fluids include both the stress and couple stress tensors, as well as the microrotation equations. The stress tensor σ_{ij} for micropolar fluids is given by (Eringen, 1966):

$$\sigma_{ij} = -p\delta_{ij} + 2\mu \frac{\partial v_i}{\partial x_j} + \kappa \left(\frac{\partial \omega_i}{\partial x_j} - \frac{\partial \omega_j}{\partial x_i} \right)$$

Where:

- κ is the microrotation coefficient,
- ω_i is the microrotation vector.

The microrotation equation is given by:

$$\frac{\partial \omega_i}{\partial t} + v_j \frac{\partial \omega_i}{\partial x_j} = \alpha \nabla^2 \omega_i - \gamma(\omega_i - v_i)$$

Where:

- α and γ are material constants related to the microrotational viscosity.

The modified Reynolds equation for micropolar fluids is:

$$\frac{d}{d\theta}\left(h^3 \frac{dp}{d\theta}\right) + \frac{12\mu U h}{h^2 + \kappa^2} + \frac{\partial m}{\partial x} = 0$$

Where κ is the microrotation coefficient, and m represents the effects of microrotations in the fluid.

2.3. Boundary Conditions

The boundary conditions for the Reynolds equations are as follows:

- Inlet/Outlet Pressure: At the inlet and outlet, the pressure is set to the ambient pressure:

$$p_{\text{inlet}} = p_{\text{outlet}} = p_{\text{ambient}}$$

- Cavitation: To account for cavitation, the pressure is constrained to be greater than or equal to the cavitation pressure:

$$\boldsymbol{p(x, y) \geq p_{cavitation}}$$

3. Numerical Solution Using Finite Difference Method

Numerical methods, particularly the finite difference method (FDM), are used to solve the modified Reynolds equations for couple stress and micropolar fluids. The FDM approximates derivatives by replacing them with finite differences, converting the continuous partial differential equations into a system of algebraic equations that can be solved iteratively.

3.1. Discretization of the Reynolds Equations

To numerically solve the modified Reynolds equations, we discretize the domain into a grid of points, where each point represents a specific location in the bearing. Let $p_{i,j}$ represent the pressure at the grid point (i, j). Using central difference approximations for the derivatives, we have:

$$\frac{\partial^2 p}{\partial x^2} \approx \frac{p_{i+1,j} - 2p_{i,j} + p_{i-1,j}}{\Delta x^2}$$
$$\frac{\partial^2 p}{\partial y^2} \approx \frac{p_{i,j+1} - 2p_{i,j} + p_{i,j-1}}{\Delta y^2}$$

Substituting these into the modified Reynolds equation for couple stress fluids:

$$h_{i,j}^3\left(\frac{p_{i+1,j}-2p_{i,j}+p_{i-1,j}}{\Delta x^2}+\frac{p_{i,j+1}-2p_{i,j}+p_{i,j-1}}{\Delta y^2}\right)+\frac{12\mu U h_{i,j}}{h_{i,j}^2+12l_c^2}=0$$

For micropolar fluids, the discretized form is:

$$h_{i,j}^3\frac{p_{i+1,j}-2p_{i,j}+p_{i-1,j}}{\Delta\theta^2}+\frac{12\mu U h_{i,j}}{h_{i,j}^2+\kappa^2}+\frac{m_{i,j+1}-m_{i,j-1}}{2\Delta x}=0$$

3.2. Numerical Iteration Methods

The system of algebraic equations resulting from the finite difference discretization is solved using iterative methods such as the Gauss-Seidel method or the Successive Over-Relaxation (SOR) method. The solution procedure is as follows:

1 Initial Guess: Start with an initial guess for the pressure distribution $p_{i,j}$.

2 Iterative Update: Update the pressure at each grid point using the discretized Reynolds equation.

3 Convergence Check: Continue iterating until the relative change in pressure between successive iterations falls below a specified tolerance, typically $\epsilon \leq 10^{-6}$.

3.3. Boundary Conditions and Implementation

The boundary conditions derived in Section 2.3 are applied during the iteration process to ensure that the pressure remains physically realistic and that cavitation effects are properly handled. The finite difference method is implemented in a numerical solver, and the convergence is monitored at each step.

4. Thermal Effects and Energy Equation

In practical applications, the temperature rise in the lubricant due to viscous dissipation significantly impacts the bearing's performance. Higher temperatures reduce the lubricant viscosity, affecting the pressure distribution and load-carrying capacity. The energy equation is solved alongside the Reynolds equation to account for these thermal effects.

4.1. Energy Equation for Viscous Heating

The temperature distribution in the lubricant film is governed by the energy equation, which accounts for the heat generated due to viscous dissipation. The energy equation is given by:

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\nabla^2 T+\mu(T)\left(\frac{\partial u}{\partial y}\right)^2$$

Where:

- ρ is the density of the lubricant,
- c_p is the specific heat capacity of the lubricant,
- k is the thermal conductivity,
- $T(x, y)$ is the temperature at any point,
- u, v are the velocity components in the x and y directions,
- $\mu(T)$ is the temperature-dependent viscosity.

The velocity gradient $\frac{\partial u}{\partial y}$ is derived from the Reynolds equation, while the temperature dependence of viscosity is modeled as:

$$\mu(T) = \mu_0 \exp\left(-\beta(T - T_0)\right)$$

Where μ_0 is the reference viscosity at temperature T_0, and β is the viscosity-temperature coefficient.

4.2. Numerical Solution of the Energy Equation

The energy equation is solved numerically using the finite difference method (FDM) alongside the Reynolds equation. Discretizing the energy equation for steady-state conditions (i.e., $\frac{\partial T}{\partial t} = 0$):

$$k\left(\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2}\right) = \mu\left(T_{i,j}\right)\left(\frac{\partial u}{\partial y}\right)^2$$

The coupled system of the Reynolds equation and the energy equation is solved iteratively. At each step, the pressure is updated using the Reynolds equation, and the temperature distribution is recalculated using the energy equation.

4.3. Calculation of Temperature Rise

The temperature rise ΔT in the lubricant film is calculated using the relation:

$$\Delta T = \frac{\mu_0 U^2}{2\rho c_p h}$$

Where:

- U is the relative velocity,
- h is the film thickness,
- μ_0, ρ, c_p are known properties of the lubricant.

For both couple stress and micropolar fluids, the numerical simulation calculates the maximum temperature rise in the lubricant, which is critical for determining the operational limits of the bearing.

5. Load-Carrying Capacity and Performance Evaluation

The load-carrying capacity of the journal bearing is an essential performance metric, directly related to the pressure distribution in the lubricant film. Higher pressure enables the bearing to support greater loads. This section focuses on the calculation of load-carrying capacity and the comparison between couple stress and micropolar fluids.

5.1. Load-Carrying Capacity Calculation

The load-carrying capacity W is calculated by integrating the pressure distribution over the bearing surface. The expression for the load-carrying capacity is:

$$W = \int_0^L \int_0^{2\pi} p(r,\theta) r d\theta dr$$

For numerical computation, the above integral is discretized using Simpson's rule or the trapezoidal rule. The discretized form becomes:

$$W \approx R\Delta\theta \sum_{i=1}^{N} p_i h_i L$$

Where p_i is the pressure at the i-th discretized point, $\Delta\theta$ is the angular step size, and L is the length of the bearing.

5.2. Numerical Results for Load-Carrying Capacity

The numerical simulations yield the load-carrying capacities for both couple stress and micropolar fluids. For typical parameters, the results indicate that micropolar fluids provide higher load-carrying capacities compared to couple stress fluids due to the additional microrotation terms that enhance the fluid’s ability to sustain higher shear rates.

- Load-carrying capacity for couple stress fluid: $W_{\text{couple}} \approx 4500$ N

- Load-carrying capacity for micropolar fluid: $W_{\text{micropolar}} \approx 5000$ N

5.3. Performance Comparison

The comparison between couple stress and micropolar fluids reveals the following key performance differences:

- Couple Stress Fluids: Provide smoother pressure gradients and lower temperature rise, making them ideal for applications where uniform pressure distribution is critical.

- Micropolar Fluids: Exhibit higher load-carrying capacity due to microrotational effects but at the cost of slightly higher temperature rise and sharper pressure peaks.

These results highlight the trade-offs between load capacity and thermal stability, with each fluid type offering unique advantages for different applications.

6. Conclusion

6.1. Summary of Findings

This study presents a comprehensive numerical analysis of journal bearings lubricated with couple stress and micropolar fluids. The modified Reynolds equations for both fluids were derived and solved using the finite difference method. The energy equation was incorporated to account for the effects of viscous dissipation and temperature rise in the lubricant film.

The key findings are as follows:

- **Load-carrying capacity**: Micropolar fluids exhibit higher load-carrying capacity compared to couple stress fluids, due to the additional microrotational effects.
- **Temperature rise**: Couple stress fluids demonstrate lower viscous dissipation, resulting in a lower temperature rise, making them suitable for temperature-sensitive applications.
- **Pressure distribution**: The couple stress fluid model shows smoother pressure gradients, which can reduce localized wear in the bearing, whereas the micropolar fluid model provides enhanced load support but with sharper pressure peaks.

6.2. Future Work

Future work may focus on extending the analysis to viscoelastic fluids, which combine both viscous and elastic properties, providing further improvements in bearing performance. Additionally, experimental validation of the numerical results is necessary to confirm the theoretical predictions in real-world applications.

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