FUZZIFICATION OF SOME PATH RELATED GRAPHS

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Abstract: *In this paper we introduce evidence labelling for fuzzification of crisp graphs. In evidence labelling, the vertex labelling is done by an injective function m and the edge labelling is done by the injective function ρ. The evidently labelled graph is a fuzzy graph. We also prove some theorems which show the admissibility of evidence labelling in path related graphs.*

Keywords: *Evidence Labelling, Path graph, Middle graph, strong duplicate graph, shadow graph, Z- graph.*

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1 INTRODUCTION

All graphs we discuss in this paper are simple and undirected.Graphs areused to explain relations between objects. To includefuzziness insuch relations fuzzy graphs were introduced. The first definition of fuzzy graph by Kaufmann was based on Zadeh's fuzzy relations. After that Rosenfield considered fuzzy relation on fuzzy sets and developed the structure of fuzzy graphs. After the work of Rosenfield, Yeh and Bang introduced various connectedness concepts of graphs and digraphs into fuzzy graphs

A labelling of a graph G is an assignment of labels to edges, vertices or both edges and vertices of a graph. In [4], A NagoorGani and D Rajalaxmi (a) Subahashini presents a new concept called fuzzy labelling and in [2] we introduced a type of labelling called fuzzy digraph labelling in fuzzy digraphs.

In this paper we introduce evidence labelling and show that a fuzzy graph can be constructed from a crisp graph by applying evidence labelling. We also Prove some theorems which shows the admissibility of evidence labelling in some path related graphs.

Definition 1

A fuzzy graph $G = (V,\mu,\rho)$ is a non empty set V together with a pair of functions.

$\mu : V \rightarrow [0,1]$ and $\rho : V \times V \rightarrow [0,1]$ such that for all x, y in V , $\rho(x, y) \leq \mu(x \wedge \mu(y)$.

We call μ the fuzzy vertex set of G and ρ , the fuzzy edge set of G respectively.

We denote the underlying graph of the fuzzy graph G = (μ,ρ) by G* = (μ*,ρ*), where μ* ={x ϵ V : μ(x) > 0} and ρ* = {(x,y) ϵ V x V : ρ(x,y) >0 }.

Definition 2

If for a pair of two paths $P_1$and P_2,i+1[th] vertex of path $P_1$is joined to the i[th] vertex of path P_2such a graph is called Z-graph.

2 MAIN RESULTS

Definition 2.1

Let G =(V,E) be a crisp graph. Then G has an evidence labelling if there exist two injections m and ρ such that m: V*→[0,1] defined by m(v_i) = $\frac{i}{N}$,where V*=$(v_1, v_{2,....,}v_N)$, $v_i \in V$and

ρ: E→[0,1] defined by

ρ(v_i, v_j)=$\frac{\mathrm{m}(\mathrm{v_i}) \wedge \mathrm{m}(\mathrm{v_j})}{N}$ $m(v_k)$ $if (v_j, v_k) \in E$ and $m(v_k) = \wedge_{(v_j,v_p)\in E}\, m(v_p)$ and $i < j < k$

and

ρ$(v_i, v_j) = \frac{\mathrm{m}(\mathrm{v_i}) \wedge \mathrm{m}(\mathrm{v_j})}{N^3}$ $if\ (v_j, v_k) \notin E\ for\ any\ k$,where$1 \leq i \leq N-1, 1 \leq j \leq N$.

The graph G which admits evidence labelling is called evidently labeled graph.

Theorem 2.1

The functionρ: E→[0,1] defined by ρ(v_i, v_j) =$\frac{\mathrm{m}(\mathrm{v_i}) \wedge \mathrm{m}(\mathrm{v_j})}{n}$ $m(v_{j+1})$, where m(v_i) = $\frac{i}{n}$is injective for monotonic m for an ordered vertex set V* suchthat v_i, v_j ∈ V*and j=i+1.

Proof

LetV* = {$\mathrm{v}_1, \mathrm{v}_2, \dots, \mathrm{v}_\mathrm{n}$}

ρ(v_i, v_j) =$\frac{\mathrm{m}(\mathrm{v_i}) \wedge \mathrm{m}(\mathrm{v_j})}{n}$ $m(v_{j+1})$

$$= \left(\frac{{}^{i}/_{n} \wedge {}^{j}/_{n}}{n}\right) {}^{(j+1)}/_{n}$$

$$= \begin{cases} \frac{i(j+1)}{n^3} if\ m(v_i) < m(v_j) \\ \frac{j(j+1)}{n^3}\ if\ m(v_i) > m(v_j) \end{cases}$$

Case I ($m(v_i) < m(v_j)$)

Then$\mathrm{m}(\mathrm{v_i}) \wedge \mathrm{m}(\mathrm{v_j})$= $\mathrm{m}(\mathrm{v_i})$ =${}^{i}/_{n}$

ρ(v_i, v_j) =$\frac{i(j+1)}{n^3}$

If possible, suppose that ρ is not injective.

Then $\rho(v_i, v_j) = \rho(v_k, v_p) \Longrightarrow \frac{i(j+1)}{n^3} = \frac{k(p+1)}{n^3}$

$$\Longrightarrow i(j+1) = k(p+1)$$

$\Longrightarrow i(i+2) = k(k+2)$ or $(j-1)(j+1) = (p-1)(p+1)$

$\Longrightarrow$ i =k, j=p.

Case II ($m(v_i) > m(v_j)$)

Then $m(v_i) \wedge m(v_j) = m(v_j) = {}^{j}/_{n}$

$\rho(v_i, v_j) = \frac{j(j+1)}{n^3}$

If possible, suppose that ρ is not injective.

Then $\rho(v_i, v_j) = \rho(v_k, v_p) \Longrightarrow \frac{j(j+1)}{n^3} = \frac{p(p+1)}{n^3}$

$$\Longrightarrow j(j+1) = p(p+1)$$

$\Longrightarrow (i+1)(i+2) = (k+1)(k+2)$ or $j(j+1) = p(p+1)$

$\Longrightarrow$ i =k, j=p.

Proposition 2.1

Every evidently labelled crisp graph is a fuzzy graph.

Proof

From the definition of evidence labelling,

$\rho(v_i, v_j) = \frac{m(v_i) \wedge m(v_j)}{N}$ $m(v_k)$ or $\frac{m(v_i) \wedge m(v_j)}{N^3} \leq m(v_i) \wedge m(v_j)$ for all i and j.

Theorem2.2

The path graph P_n admits evidence labelling.

Proof

Consider the path graph P_n, where $|V(G)|$ = n , $|E(G)|$= n-1.

Let $V^* = (v_1, v_2, \ldots, v_n)$.

Define m : $V^* \rightarrow [0,1]$ as $m(v_i) = \frac{i}{n}$, $1 \leq i \leq n$ and

ρ: $E \rightarrow [0,1]$ as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{n^3}$, $1 \leq i \leq n-1$.

$\rho(v_{n-1}, v_n) = \frac{(n-1)}{n^4}$

If possible, suppose that m is not injective.

Then $m(v_i) = m(v_j)$ for some $i \neq j$

$\Longrightarrow$ i =j.

If possible ,suppose that $\rho(v_i, v_{i+1}) = \rho(v_j, v_{j+1})$ for some $i \neq j$

$\Longrightarrow \frac{i(i+2)}{n^3} = \frac{j(j+2)}{n^3}$

$\Longrightarrow i^2 - j^2 = 2(j-i)$

$\Longrightarrow i + j = -2$, contradiction.

Also, $\frac{(n-1)}{n^4} < \frac{i(i+2)}{n^3}$ for all i.

Theorem2.3

The square of path graph P_n admits evidence labelling.

Proof

Consider the square of P_n, $P_n^2=(V,E)$ where $|V| = n$,$|E|=2n-3$

Let $V^* = (v_1, v_2, \ldots, v_n)$.

Define m : $V^* \to [0,1]$ as $m(v_i) = \frac{i}{n}$, $1 \leq i \leq n$

ρ: E →[0,1] as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{n^3}$, $1 \leq i \leq n-2$.

$\rho(v_{n-1}, v_n) = = \frac{n-1}{n^4}$.

$\rho(v_i, v_{i+2}) = \frac{i(i+3)}{n^3}$, $1 \leq i \leq n-3$.

$\rho(v_{n-2}, v_n) = = \frac{n-2}{n^4}$

Clearly mand ρ are injective.

Theorem 2.4

The middlegraph of path graph P_n admits evidence labelling.

Proof

Consider the middle graph $M(P_n)$ of P_n , where $|V| = 2n-1$,$|E| = 3n-4$.

Let $V^* = (v_1, v_2, \ldots, v_n, v_{n+1} = e_1, v_{n+2} = e_2, \ldots, v_{2n-1} = e_{n-1})$, $e_i = (v_i, v_{i+1})$, v_i is a vertex in P_n.

Define m : $V^* \to [0,1]$ as $m(v_i) = \frac{i}{2n-1}$, $1 \leq i \leq 2n-1$ and

ρ:E→[0,1] as $\rho(v_i, v_{i+n}) = \frac{i(i+1)}{(2n-1)^3}$, $1 \leq i \leq n-2$

$\rho(v_{n-1}, v_{2n-1}) = \frac{n-1}{(2n-1)^4}$.

$\rho(v_{i+1}, v_{i+n}) = \frac{(i+1)(i+n+1)}{(2n-1)^3}$,$1 \leq i \leq n-2$.

$\rho(v_n, v_{2n-1}) = \frac{n}{(2n-1)^4}$.

$\rho(v_{i+n}, v_{i+n+1}) = \frac{(i+n)(i+n+2)}{(2n-1)^3}$, $1 \leq i \leq n-3$.

ρ(v_{2n-2}, v_{2n-1}) =$\frac{(2n-2)}{(2n-1)^4}$.

Clearly m and ρ are injective.

Theorem 2.5

The strong duplicategraph of path graph P_n, admits evidence labelling.

Proof

Consider the strong duplicate graph SD(P_n) =(V,E) of P_n , where $|V|$ = 2n ,$|E|$= 3n-2.

Let V* =(v_1,v_2,....v_n, v_{n+1}, v_{n+2},... v_{2n}), v_i= v_{n+i} for all i.

Define m : V*→[0,1] as m(v_i) =$\frac{i}{2n}$, $1 \le i \le 2n-1$ and

ρ: E →[0,1] asρ(v_i, v_{i+n+1}) = $\frac{i}{(2n)^4}$,$1\le i \le n-1$

ρ(v_{i+1}, v_{i+n}) = $\frac{(i+1)}{(2n)^2}$, $1\le i \le n-2$.

ρ(v_n, v_{2n-1}) =$\frac{n}{(2n)^4}$.

ρ(v_i, v_{i+n}) = $\frac{i(i+1)}{(2n)^3}$, $1\le i \le n-1$

ρ(v_n, v_{2n}) =$\frac{n}{(2n)^4}$.

Clearly m and ρ are injective.

Theorem2.6

The shadow graph of path graph P_n admits evidence labelling.

Proof

Consider the shadow graphD_2(P_n)= (V,E) of path graph P_n , where$|V(G)|$ = 2n ,

$|E(G)|$= 4n -4.

Let V* =(v_1,v_2,...,v_n,v_{n+1},v_{n+2},...v_{2n}), v_i= v_{n+i} for all i.

Define m : V*→[0,1] as m(v_i) =$\frac{i}{2n}$, $1 \le i \le 2n$

ρ: E →[0,1] asρ(v_i, v_{i+1}) = $\frac{i(i+2)}{(2n)^3}$, $1 \le i \le n-2$.

ρ(v_{n-1},v_n) = $\frac{(n-1)(2n-1)}{(2n)^3}$.

ρ(v_i, v_{i+n+1}) = $\frac{i(i+n+2)}{(2n)^3}$,$1 \le i \le n-2$. .

ρ(v_{i+1}, v_{i+n}) = $\frac{(i+1)(i+n+1)}{(2n)^3}$, $1 \le i \le n$.

ρ(v_{i+n}, v_{i+n+1}) = $\frac{(i+n)(i+n+2)}{(2n)^3}$, $1 \le i \le n-2$. .

$\rho(v_{2n-1},v_{2n}) = \frac{2n-1}{(2n)^4}$

Clearly m and ρ are injective.

Theorem 2.7

The corona of a path graph P_n admits evidence labelling.

Proof

Consider the corona G= $P_n \odot K_1$ = (V,E) of path graph P_n, where $|V(G)|$ = 2n , $|E(G)|$= 2n -1.

Let $V^* = (v_1,v_2,\dots,v_n,v_{n+1},v_{n+2},\dots v_{2n})$.

Define m : $V^* \to [0,1]$ as $m(v_i) = \frac{i}{2n}$, $1 \le i \le 2n$

ρ: E →[0,1] as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{(2n)^3}$, $1 \le i \le n-2$.

$\rho(v_{n-1},v_n) = \frac{(n-1)2n}{(2n)^3}$.

$\rho(v_i, v_{i+n}) = \frac{i}{(2n)^4}$, $1 \le i \le n$.

Clearly m and ρ are injective.

Theorem 2.8

The H-graph of a path graph P_n admits evidence labelling.

Proof

Consider the H-graph G= (V,E) of path graph P_n, where $|V(G)|$ = 2n , $|E(G)|$= 2n -1.

Let $V^* = (v_1,v_2,\dots,v_n,v_{n+1},v_{n+2},\dots,v_{2n})$.

Define m : $V^* \to [0,1]$ as $m(v_i) = \frac{i}{2n}$, $1 \le i \le 2n$

Case I-when n is even

Without loss of generality assume that $(v_{\frac{n}{2}},v_{\frac{3n+2}{2}})$is an edge.

Define ρ: E →[0,1] as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{(2n)^3}$, $1 \le i \le n-2$, $n+1 \le i \le 2n-2$.

$\rho(v_{n-1},v_n) = \frac{(n-1)}{(2n)^4}$.

$\rho(v_{\frac{n}{2}},v_{\frac{3n+2}{2}}) = \frac{n(3n+4)}{4(2n)^4}$.

Clearly m and ρ are injective.

Case II - when n is odd

Without loss of generality assume that $(v_{\frac{n+1}{2}},v_{\frac{3n+1}{2}})$is an edge.

Define ρ: E →[0,1] as ρ(v_i, v_{i+1}) = $\frac{i(i+2)}{(2n)^3}$, $1 \leq i \leq n-2$, n+1 $\leq i \leq 2n-2$

ρ(v_{n-1},v_n) = $\frac{(n-1)}{(2n)^4}$.

ρ($v_{\frac{n+1}{2}}$,$v_{\frac{3n+1}{2}}$) = $\frac{n(3n+3)}{4(2n)^4}$.

Clearly m and ρ are injective.

Theorem 2.9

The Z-graph of a path graph P_n admits evidence labelling.

Proof

Consider the Z-graph G= (V,E) of path graph P_n , where $|V(G)|$ = 2n ,

| $E(G)$|= 3n -3.

Let V* =(v_1,v_2,...v_n,v_{n+1},v_{n+2},...v_{2n}).

Define m : V*→[0,1] as m(v_i) =$\frac{i}{2n}$, $1 \leq i \leq 2n$and

ρ: E →[0,1] as ρ(v_i, v_{i+1}) = $\frac{i(i+2)}{(2n)^3}$, $1 \leq i \leq n-2$, n+1 $\leq i \leq 2n-2$.

ρ(v_{n-1},v_n) = $\frac{(n-1)(2n-1)}{(2n)^3}$.

ρ(v_{2n-1},v_{2n}) = $\frac{(2n-1)}{(2n)^4}$.

ρ(v_{i+1}, v_{i+n}) = $\frac{(i+1)(i+n+1)}{(2n)^3}$, $1 \leq i \leq n-1$.

Clearly m and ρ are injective.

CONCLUSION

In this paper we introduce evidence labelling using two injective functions for the fuzzification of crisp graphs. We prove some theorems which show the admissibility of evidence labelling in path related graph s like shadow graph, middle graph, strong duplicate graph etc.

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