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ANALYTICAL RELATION & COMPARISON OF PSNR AND SSIM ON BABBON IMAGE AND HUMAN EYE PERCEPTION USING MATLAB

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Abstract: *In this paper we conduct a analytical relation & comparison of PSNR and SSIM on babbon image and human eye perception using MATLAB. The measures have been categorized into pixel difference-based, and HVS-based (Human Visual System-based) measures. It has been found that measures based on HVS, on phase spectrum and on multi resolution mean square error are most discriminative to coding artifacts.In recent researches of image processing, a research has been done to measure the quality of image. The image quality assessment plays an important role where the quality is to be assessed after or before using the image for any purpose. Pixel difference-based are calculated pixel wise. As far as HVS is concerned it checks the luminosity, contrast and structure in image. Ahead in this article it will be discussed that how human visual system (HVS) is preferred over other system in an approach of full reference objective quality metrics. A comparative study between these objective quality metrics technique is always an interesting topic.*

Key Words: *Pixel difference Based measurement, Mean squared error (MSE), Peak signal-to-noise ratio (PSNR) , Human Visual System based (HVS), Measurement, Image quality measurement, Structural similarity index metric (SSIM), Universal Image Quality Index (UIQI)*

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I. INTRODUCTION

An objective image quality metric approach plays vital roles in the applications of image processing. The first role is to monitor and adjust the image quality dynamically. Optimization of algorithm and the parameter setting of image processing system is second role. While the third role is to benchmark image processing systems and its algorithm[9]. The approach of Objective image quality metric is classified into three approaches, First and most exciting is full reference. In this approach the test image is compared with a known available reference image. In second approach i.e. in no reference or blind reference image quality assessment there is no image available which can be compared with the test image. This approach is found in most practical applications. While in third approach, the reference image is partially available to be compared with the test image[9]. The availability of the partially available reference image is generally in form of a set of extracted features. This approach is termed as reduced-reference image quality metrics.

A bivariate or full reference image quality metrics can be approached by following ways:

1. Pixel difference-based measures such as mean square distortion.
2. Correlation-based measures, that is, correlation of pixels, or of the vector angular directions.
3. Edge-based measures, that is, displacement of edge positions or their consistency across resolution levels.
4. Spectral distance-based measures, that is Fourier magnitude and/or phase spectral discrepancy on a block basis.
5. Context-based measures, that is penalties based on various functional of the multidimensional context probability.
6. Human Visual System-based measures, measures either based on the HVS weighted spectral distortion measures or (dis)similarity criteria used in image database browsing functions.

This paper is focuses on only Pixel Difference-based measures and Human Visual System-based measures.

II. PIXEL DIFFRENCE-BASED MEASURES

In this Measurement technique two images are taken one of them is reference image and other is test image whose image quality is to be assessed. A sum of an undistorted reference signal and an error signal helps in evaluation of quality of image signal. Commonly used Pixel Difference-based measures are:

1. Mean Squared Error(MSE)
2. Peak Signal to Noise Ratio(PSNR)

These measures are illustrated are as follows:

Mean Squared Error is discussed as a signal fidelity measure. The approach of signal fidelity measure is to compare two signals by providing a quantitative score that describe the degree of similarity/fidelity or, conversely, the level of error distortion between them. Usually, it is assumed that one of the signals is a pristine original, while the other are distorted or contaminated by errors [2].

Let us assume that X = $\{x_i | i = 1, 2 \ldots N\}$ and Y = $\{y_i | i = 1, 2 \ldots N\}$ are two infinite length, discrete signals (this discrete signal is considered as a visual signal), where N is the number of pixels in digital image and x_i and y_i are the values of i^{th} pixel of the digital image X and digital image Y respectively. The MSE between the digital images can be expressed mathematically as:

$$\mathrm{MSE}(X,\ Y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - y_i)^2 \qquad (1)$$

In MSE, the square of the error signal is commonly referred; error signal is given by the difference between the whole infinite lengths of pixels of digital images given by $e_i = x_i - y_i$. This difference is the difference between reference digital image and distorted test image. In general form, in l_p norm the MSE can be expressed as:

$$d_p(x,y) = \left(\sum_{i=1}^{N} |e_i|^p\right)^{1/p} \qquad (2)$$

The reason behind MSE is so popular is that, it is simpler to compute. MSE is parameter free so the complexities of only one multiply and two additions per pixel. As it is memory less so the computed squared error of pixels are independent of other sample. Energy of error is defined in such a way that its measures are preserved after any orthogonal or unitary linear transformation. The energy preserving property makes sure that the energy of a signal distortion in the transform domain is same as computed in signal domain.

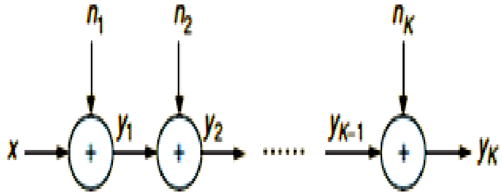


Fig. 1 Independent additive sources of distortions and additive property of the MSE

Fig. 1 illustrates that the MSE is additive for independent source distortions, where a zero-mean random source x passes through a cascade of K additive independent zero-mean distortion $n_1, n_2 \ldots n_K$, resulting in $y_1, y_2 \ldots y_K$, mathematically;

$$y_k = x + \sum_{i=1}^{k} n_i \quad \text{for } k = 1, 2 \ldots K \qquad (3)$$

The sum of the MSEs from the individual distortion images results in the overall MSE of the image.

$$\text{MSE}(X,Y) = E[(x - y_k)^2]$$

$$= E\left[\left(\sum_{k=1}^{K} n_k\right)^2\right]$$

$$= \sum_{k=1}^{K} E\,[n_k^2]$$

$$= \text{MSE}\,(x, y_1) + \text{MSE}\,(y_1, y_2) + \ldots + \text{MSE}\,(y_{k-1}, y_k)$$

The main issue for which MSE becomes unfavorable is that the readings taken with the help of MSE measures may or may not meet the specification of human eye perception. In technical terms two images with similar MSE can have different eye perception. The two Images having same MSE out of them one can be viewed better while other cannot even be identified.

Peak Signal-to-Noise Ratio is the ratio between the reference signal and the distortion signal in an image, given in decibels[5]. The higher the PSNR, the closer the distorted image is to the original. In other words we can say it is the reciprocal of the decibel scale of MSE. So we can say for a higher image quality if MSE approaches to zero the PSNR of the image will approach to infinity. In general, a higher PSNR value should correlate to a higher quality image, but tests have shown that this isn't always the case.

Let us assume that X = $\{x_i | i = 1, 2 \ldots N\}$ and Y = $\{y_i | i = 1, 2 \ldots N\}$ are two infinite length, discrete signals (this discrete signal is considered as a visual signal), where N is the number of pixels in digital image and x_i and y_i are the values of i^{th} pixel of the digital image X and digital image Y respectively. Mathematically, the PSNR for the full reference Image quality metrics is given by:

$$PSNR(X,\ Y) = 10\, log_{10}\left(\frac{MPP^2}{MSE(X,\ Y)}\right) \qquad (4)$$

Where,

MPP is Maximum Possible Pixel in an image, i.e. if the image of 8 bit then the MPP = $2^8 - 1$ = 255 pixels.

MSE(X, Y) is the Mean Square error of the image X and Image Y.

III. HUMAN VISUAL SYSTEM-BASED MEASURES

In recent researches for image quality assessment a major emphasis is on understanding and analyzing the Human Visual System features. According to the titans in research field of image processing it is believed that the accuracy can be increased more if the perception of human eye is incorporated into Objective Image Quality Metrics System. In the recent decades most dominant paradigm is HVS-based FR paradigm. It is because the human do not perceive images in high-dimensional space, but are interested in different attributes of image, like brightness, contrast, shape, texture of object, orientation, smoothness, etc. HVS systems always adopt different aspect for different image according to the sensitivity of system. Human Visual System (HVS) has been extensively exposed to the natural visual environment, and a variety of evidence has shown that the HVS is highly adapted to extract useful information from natural scenes. Two Human visual systems (HVS) based image quality measures are given below:-

1. Universal Image Quality Index (UIQI)
2. Structural Similarity Index Metric (SSIM).

Universal Image Quality Index (UIQI): Let us assume that X = $\{x_i | i = 1, 2 \ldots N\}$ and Y = $\{y_i | i = 1, 2 \ldots N\}$ are two infinite length, discrete signals (this discrete signal is considered as a visual signal), where N is the number of pixels in digital image and x_i and y_i are the values of i^{th} pixel of the digital image X and digital image Y respectively.

Then,

$$UIQI = \frac{4 \times \sigma_{xy} \times \bar{x} \times \bar{y}}{(\sigma_x^2 + \sigma_y^2) \times ((\bar{x})^2 + (\bar{y})^2)} \qquad (5)$$

Where,

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{6}$$

$$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i \tag{7}$$

$$\sigma_x^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2 \tag{8}$$

$$\sigma_y^2 = \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})^2 \tag{9}$$

$$\sigma_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y}) \tag{10}$$

The dynamic range of UIQI is [0, 1][6]

Best value UIQI=1, is achieved when $y_i = x_i$, i = 1, 2 …N.

This quality index models any distortion as a combination of three different factors: loss of correlation, luminance distortion, and contrast distortion. In order to understand this, rewriting the definition of Q as a product of three components:

$$UIQI = Q_1 \times Q_2 \times Q_3 \tag{11}$$

$$Q_1 = \frac{\sigma_{xy}}{\sigma_x \times \sigma_y} \tag{12}$$

$$Q_2 = \frac{2 \times \bar{x} \times \bar{y}}{(\bar{x})^2 + (\bar{y})^2} \tag{13}$$

$$Q_3 = \frac{2 \times \sigma_x \times \sigma_y}{(\sigma_x^2 + \sigma_y^2)} \tag{14}$$

$$Q = \frac{\sigma_{xy}}{\sigma_x \times \sigma_y} \times \frac{2 \times \bar{x} \times \bar{y}}{(\bar{x})^2 + (\bar{y})^2} \times \frac{2 \times \sigma_x \times \sigma_y}{(\sigma_x^2 + \sigma_y^2)} \tag{15}$$

Q_1 measures the degree of linear correlation between *x* and *y*. But if *x* and *y* are linearly related then still there might be relative distortion between them. The Q_2 and Q_3 components evaluates this relative distortion between the image X and image Y. the Q_2measures the closeness of luminance between the two images while Q_3 estimates the similarities between the two contrast of the image X and Y.

Both of Q_2 and Q_3 range the value [0, 1], where the best value 1 is achieved if and only if $\sigma_x = \sigma_y$.

The overall Universal image quality index is the average of all the UIQI's value in the quality map:

$$UIQI = \frac{1}{M}\sum_{k=1}^{M} UIQI_k \tag{16}$$

Structural Similarity Index Metric (SSIM): The product of illumination and the reflectance is the luminance of object that is observed by the eye of observer. Whereas the structure of the object is concerned it is free from luminance. The structural information in an image can be defined as the attributes, represents the structure of the object in image, independent of average luminance and contrast of image [9]. For the two non negative image signals X and Y, if we consider one is reference image and other is test image, then the similarity measure can serve as a quantitative measurement of quality of second image.

The system shown in Fig 2 separates the task of similarity measurement into following three comparisons:

i. Luminance Comparison
ii. Contrast Comparison
iii. Structure Comparison

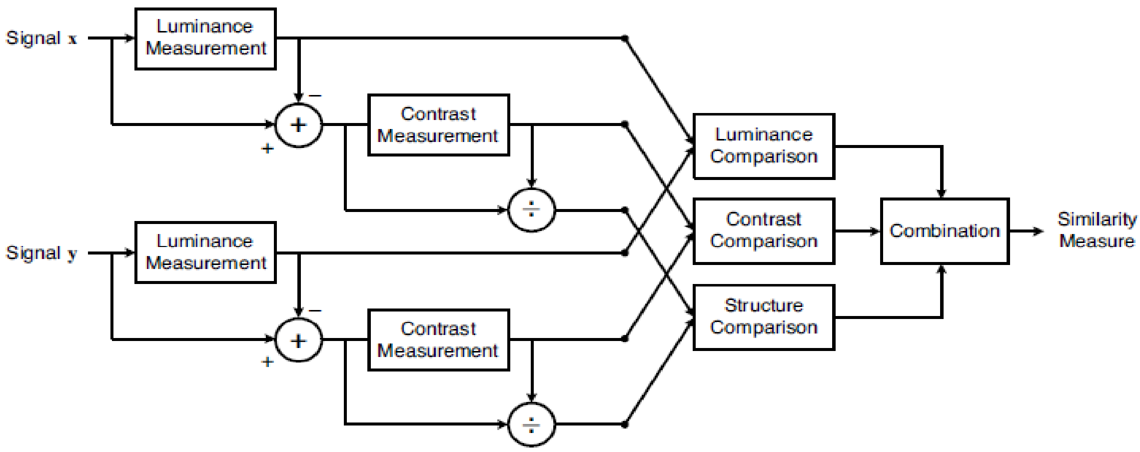


Fig 2 SSIM measurement system diagram

The luminance of each signal is compared by assuming discrete signals and is estimated as the mean intensity.

$$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad (17)$$

The luminance comparison function *l(x, y)* is then function of $\mu_x$and μ_y.

The Contrast comparison is estimated by removing the mean intensity from the signal. In discrete form, the resulting signal x - μ_x corresponds to the projection of vector *x* onto the hyper plane defined by:

$$\sum_{i=1}^{N} x_i - 0 \qquad (18)$$

An unbiased estimate in discrete form is given by:

$$\sigma_x = \sqrt[2]{\left(\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu_x)^2\right)} \qquad (19)$$

Then the contrast comparison c(x, y) is then the comparison of σ_x and σ_y.

$(x-\mu_x)/\sigma_x$and$(x-\mu_x)/\sigma_x$are two normalized signals on which the structure comparison *s(x, y)* is conducted.

And at last these three comparisons are yields an overall similarity measure given by:

$$S(x, y)= f(l(x, y), c(x, y), s(x, y)) \qquad (20)$$

From (20) it can be said that the three components in the image are relatively independent in this system. And is we have to define the functions *l(x, y), c(x, y)* and *s(x, y)* as well as the combination function *f(.)* in order to define the similarity measure completely as in (20).

For luminance comparison, we define

$$l(x,y) = \frac{2\mu_x\mu_y+C_1}{\mu_x^2+\mu_y^2+C_1} \qquad (21)$$

Where the constant C_1 is included to avoid instability when $\mu_x^2+\mu_y^2$ is very close to zero. Specifically, we chose

$$C_1 = (K_1 MPP)^2 \qquad (22)$$

where, MPP is the maximum possible pixel in a grayscale image, and $K_1 << 1$ is small constant. For the contrast and structure comparison same consideration is done as done in (21).

Weber's Law, widely used to model light adaptation in the HVS, is also qualitatively consistent with (6). According to this law the magnitude of a just noticeable luminance change Δ*I* is approximately proportional to the background luminance *I* for a wide range of luminance values. Letting *R* represents the size of luminance change relative to background luminance, we rewrite the luminance of the distorted signal as $\mu_y = (1+R)\mu_x$. Substituting this into (21) results in

$$l(x,y) = \frac{2(1+R)}{1+(1+R)^2+\frac{C_1}{\mu_x^2}} \qquad (22)$$

If C_1 is assumed is small enough to be ignored, then *l(x, y)* is a function only of *R.*

The contrast comparison function takes a similar

$$c(x,y) = \frac{2\sigma_y+C_2}{\sigma_x^2+\sigma_y^2+C_2} \qquad (23)$$

Where, $K_2<<1$. The important feature of the function is that with same amount of contrast change $\Delta\sigma = \sigma_y - \sigma_x$, this measure is less sensitive to case of high base contrast σ_x than low bas contrast. This is consistent with the contrast-masking feature of the HVS.

Similarly Structure comparison function can be defined as follows:

$$s(x,y) = \frac{\sigma_{xy} + C_3}{\sigma_x \sigma_y + C_3} \quad (24)$$

As in luminance and contrast measures, a small constant in numerator and denominator is introduced. σ_{xy} can be estimated in discrete form as:

$$\sigma_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu_x)(y_i - \mu_y) \quad (25)$$

After combining the equation (22), (23) and (24) the final equation for the SSIM index between image X and Y is determined:

$$SSIM\ (x,y) = [l(x,y)]^{\propto}.[c(x,y)]^{\beta}.[s(x,y)]^{\gamma} \quad (26)$$

Where, α, β and γ must be greater than zero are parameters used to adjust the relative importance of the three components. For the simplification of calculation α = β = γ =1 and $C_3 = C_2/2$ is taken here. From the given assumption SSIM index is:

$$SSIM\ (x,y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_x\sigma_y + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \quad (27)$$

In UIQI special case generally $C_1 = C_2 = 0$ is defined resulting the unstable result to be unstable [9]. The relationship shows between the SSIM index and more traditional quality metrics illustrated geometrically in a vector space of image components. These image components can be either pixel intensities or other extracted features such as transformed linear coefficient.

IV. ANALYTICAL RELATION BETWEEN MSE, PSNR AND SSIM

As the relation between MSE and PSNR is already discussed in this paper so now the focus on MSE and SSIM will be given and they will be related with each other by the given equations.

The MSE in equation (1) can be redefined as:

$$MSE = \sigma_x^2 + \sigma_y^2 - 2\sigma_{xy} + (\mu_x + \mu_y)^2 \quad (28)$$

Where σ_x^2 and σ_y^2 are the variance of image X and Y and σ_{xy} is covariance between image X and Y.

So the SSIM defined in equation (27) can be written as:

$$\frac{1}{SSIM} = \frac{MPP^2 \times \alpha(x,y) \times e^{-PSNR\ \times log\ 10\ 10} + \beta(x,y)}{l(x,y)s(x,y)} \quad (29)$$

Where,

$$\alpha(x,y) = \frac{1}{2\sigma_x\sigma_y C_2}$$

$\beta(x,y) = \frac{2\sigma_{xy} - (\mu_x - \mu_y)^2 + C_2}{2\sigma_x\sigma_y C_2}$

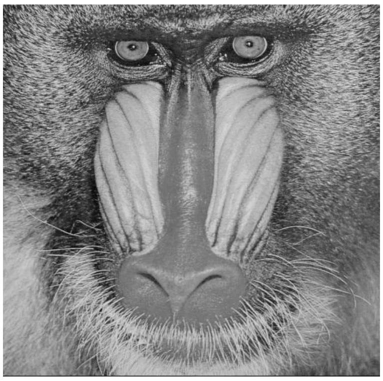
Let us assume $C_2 << \sigma_x, \sigma_y$ and $C_3 << \sigma_x, \sigma_y$. This assumption is made to nullify the effect of the constants appearing in the SSIM formula. Thus, in the case of non-null standard deviation values, the constants can be discarded. Non-null standard deviation values are found in real images on which at least one pixel has a grey-level value different from the other pixels. In such case PSNR can be defined in term as:

$$PSNR = log_{10}\left[\frac{2\sigma_{xy}\,(l(x,y) - SSIM)}{MPP^2 SSIM} + \left(\frac{\mu_x - \mu_y}{MPP}\right)^2\right] \qquad (30)$$

The relationship described in (30) is general and can be used for any kind of image degradation. This relationship can be further simplified in the case of some common image degradation [3].

V. MSE PSNR AND SSIM ON AN IMAGES

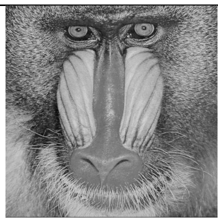
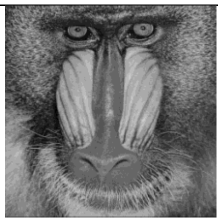
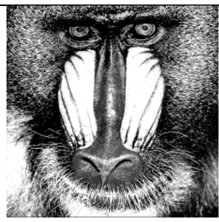
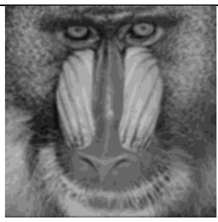
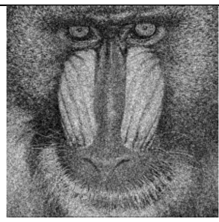
In this paper, Baboon Image has been used as standard images. Four operations, compression, Contrast addition, Gaussian noise and Blur, is being added to the standard image and then MSE, PSNR and SSIM algorithm is applied on the images for full reference.



Baboon Image

Using MATLAB the algorithms for MSE, PSNR and SSIM are applied on the above shown images for the image quality assessment and following result is deduced. This result is than compared with each other and also with the human eye perception so that the better assessment technique can be rated best.

Test on Baboon Image

<table>
<tr><th colspan="2">Operation & Result</th><th colspan="2">Image with Noise</th></tr>
<tr><td colspan="2">Original
MSE=0
PSNR=∞
SSIM=1</td><td colspan="2"></td></tr>
<tr><th>Operation & Result</th><th>Image with Noise</th><th>Operation & Result</th><th>Image with Noise</th></tr>
<tr><td>Compressed Image
MSE=35.4033
PSNR=40.3857
SSIM=0.8718(1st)</td><td></td><td>Contrast Addition
MSE=3.0663
PSNR=45.6977
SSIM=0.8324(2nd)</td><td></td></tr>
<tr><td>Blurred image
MSE=30.0145
PSNR=40.7441
SSIM=0.7606(3rd)</td><td></td><td>Gaussian Noise
MSE=49.2452
PSNR=39.6689
SSIM=0.1360(4th)</td><td></td></tr>
</table>

VI. CONCLUSION

Image Quality Assessment plays a vital role in Image processing. Lot of research work is done in this section also in order to reduce the time, complexity and cost to assess the quality of image. Relation between various technique helps in getting the result form previous technique to new technique with the help of some mathematic. The aim to show comparison between the various techniques is just to bench mark the better algorithm for the Image quality metric. A comparison between these two popular assessment techniques i.e. Pixel difference-based measurement metrics and HVS based metric shows that, of course calculating the image quality metric with the help of MSE or PSNR is very simple as compared to UIQI or SSIM. But the results which satisfy the need of human eye are provided by Structural Similarity (SSIM).

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