FUNDAMENTAL THEORY OF SEQUENCES IN INTUITIONISTIC FUZZY SOFT TOPOLOGICAL SPACE

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Abstract: *Topology and sequence are important mathematical tools for exploratory data mining and common techniques for statistical data analysis, used in many fields, including machine learning, pattern recognition, image analysis, information retrieval and bioinformatics. In this research work, we introduce new types of intuitionistic fuzzy soft sequences and their fundamental properties in intuitionistic fuzzy soft topological spaces are studied. The concepts of subsequence, increasing sequence, decreasing sequence and convergence sequence of intuitionistic fuzzy soft sets are proposed. The concepts of clusters of intuitionistic fuzzy soft sequences are also introduced. Some new results regarding the above concepts are explored.*

Keywords: *Soft set, fuzzy set, fuzzy soft set, intuitionistic fuzzy soft set, intuitionistic fuzzy soft topological space, intuitionistic fuzzy soft sequence.*

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I. INTRODUCTION

The uncertainty, vagueness or the representation of imperfect knowledge has been a problem in artificial intelligence, network and communication; signal processing, machine learning, computer science, information technology, medical science, economics, environments, engineering, many others. There are many mathematical tools for dealing with uncertainties; some of them are fuzzy set theory [11] and soft set theory [7]. Maji et al. [5] defined several operations on soft set theory. Established on the analytic thinking of various operations on soft sets defined in [7], Ali et al. [1] motivated some new algebraic operations on soft sets and tried out that certain De Morgan's law holds in soft set theory with regard to these new definitions. Combining soft sets [7] with fuzzy sets[11] and intuitionistic fuzzy sets [2], Maji et al. [4, 6] defined fuzzy soft sets and intuitionistic fuzzy soft sets, which are rich potential for solving decision making problems. In 2011 Shabir and Naz [8] defined soft topology by using soft sets and presented basic properties in their paper. Tanay and other [9, 10] defined fuzzy soft topology on a fuzzy soft set over an initial universe. Li and Cui [3] defined the topological structure of intuitionistic fuzzy soft sets taking the whole parameter set E.

In this paper, we have introduced new types of intuitionistic fuzzy soft sequences and study their basic properties in intuitionistic fuzzy soft topological spaces, taking the whole parameter set E. The concepts of subsequence, convergence sequence and a cluster of intuitionistic fuzzy soft sequence are proposed. Some new results regarding the above concepts are also defined.

Now, we give some ready reference for our further discussion.

A. Soft set [9]

Let U be an universe set and E be a set of parameters. Let P(U) denotes the power set of U and A$\subseteq$E. Then the pair (F, A) is called a soft set over U, where F is a mapping given by F: A$\to$ P(U).

B. Intuitionistic Fuzzy Set[2]

Let U be a non empty set. Then an intuitionistic fuzzy set (IF set for short) A on U is a set having the form $\mathrm{A} = \{\langle x, \mu_{\mathrm{A}}(x), \nu_{\mathrm{A}}(x)\rangle : x \in U\}$ where the functions μ_{A} : U $\to$[0, 1] and ν_{A} : U

$\rightarrow$[0, 1] represents the degree of membership and the degree of non-membership respectively of each element x$\in$U and $0 \le \mu_A(x) + \nu_A(x) \le 1$, for each x$\in$U.

C. Intuitionistic Fuzzy Soft Set[6]

Let U be an universe set and E be a set of parameters. Let IF(U) be the set of all intuitionistic fuzzy subsets of U and A$\subseteq$E. Then the pair (F, A) is called an intuitionistic fuzzy soft set (in short IF-soft set) over U, where F is a mapping given by F: A$\rightarrow$ IF(U).

Li and Cui **[3]** denote the IF-soft set (F, A) as F_A and the set of all IF-soft sets over U with fixed parameter E is denoted by $IFS(U)_E$.

D. IF-soft subset[6]

Let F_A and G_B be two IF-soft sets over (*U*, *E*). Then F_A is an IF-soft subset of G_B, if $A \subseteq B$ and for all $e \in A,\ F(e) \subseteq G(e)$. We write $F_A \tilde{\subseteq} G_B$.

E. Intersection[6]

The intersection of two IF-soft sets F_A and G_B over (*U*, *E*) is an IF-soft set H_C where $C = A \cap B$ and $\forall e \in C,\ H(e) = F(e) \cap G(e)$ and is written as $F_A \tilde{\cap} G_B = H_C$ (here $\cap$ represents the intuitionistic fuzzy intersection).

F. Union[6]

The union of two IF-soft sets F_A and G_B over (*U*, *E*) is an IF-soft set H_C where $C = A \cup B$ and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & if\ \ e \in A - B \\ G(e), & if\ \ e \in B - A \\ F(e) \cup G(e), & if\ \ e \in A \cap B \end{cases}$$

and is written as $F_A \tilde{\cup} G_B = H_C$ (here $\cup$ represents the intuitionistic fuzzy union)

G. Complement[6]

The complement of an IF-soft set *(F, A)* over (*U*, *E*) is denoted by $(F,\ A)^C$ and is defined by $(F,\ A)^C = (F^C,\ A)$, where $F^C : A \rightarrow IF(U)$ is a mapping given by $F^C(e) = (F(e))^C, \forall e \in A$, where c is IF-complement.

We denote the complement of an IF-soft set *(F, A)* by $(F_A)^C$.

H. IF-soft topological space[3]

Let $\tau \subseteq IFS(U)_E$, then τ is called an IF-soft topology on U if the following conditions are satisfied:

$[O_1]$. U_E, $\phi_E \in \tau$, (where U_E *and* ϕ_E defined in **[3]**),

$[O_2]$. $F_E, G_E \in \tau \Rightarrow F_E \tilde{\cap} G_E \in \tau$,

$[O_3]$. $\{(F_\alpha)_E \mid \alpha \in \Gamma\} \subseteq \tau \Rightarrow \tilde{\cup}_{\alpha \in \Gamma} (F_\alpha)_E \in \tau$.

The triplet (U, τ, E) is called an IF-soft topological space over U. Every member of τ is called an IF-soft open set.

I. Neighbourhood and Neighbourhood System[3]

Let (U, τ, E) be an IF-soft topological space and $F_E \in IFS(U)_E$. Then

1. $G_E \in IFS(U)_E$ is a neighbourhood (in short nbd) of F_E iff $\exists\, H_E \in \tau$ such that $F_E \tilde{\subseteq} H_E \tilde{\subseteq} G_E$.
2. The family of all nbds of F_E is called the neighbourhood system of F_E.

J. Proposition [3]

$F_E \in IFS(U)_E$ is an open iff F_E is a neighbourhood of each $G_E \in IFS(U)_E$ contained in F_E.

II. MAIN RESULTS

In this section we introduce new types of IF-soft sequences and study their fundamental properties in an IF-soft topological space.

A. IF-soft sequence

Let (U, τ, E) be the IF-soft topological space over U and N be the set of all natural numbers. An IF-soft sequence in (U, τ, E) is a mapping from N to $IFS(U)_E$ and is denoted by $\{(F_n)_E\}$ *or* $\{(F_n)_E : n = 1, 2, 3, \ldots\}$.

Example 1

Let $U = \{h_1, h_2, h_3, h_4\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$ and $A = \{e_1, e_2, e_3\}$.

If we chose for n=1, 2, 3….

$$\begin{aligned}(F_n)_E = \{ e_1 &= \{\langle h_1,(0.1,2/5n)\rangle,\langle h_2,(1/8n,0.3)\rangle,\\ &\quad \langle h_3,(1/5n,0.5)\rangle,\langle h_4,(0,2/5n)\rangle\}\\ e_2 &= \{\langle h_1,(0.1,0.3)\rangle,\langle h_2,(1/8n,3/5n)\rangle,\\ &\quad \langle h_3,(2/7n,3/5n)\rangle,\langle h_4,(0,0.5)\rangle\}\\ e_3 &= \{\langle h_1,(1/8n,0.5])\rangle,\langle h_2,(0.2,1/2n)\rangle,\\ &\quad \langle h_3,(0.1,2/5n])\rangle,\langle h_4,(1/5n,0.3)\rangle\},\end{aligned}$$

then $\{(F_n)_E : n = 1,2,3...\}$ forms an IF-soft sequence.

B. Eventually contained

An IF-soft sequence $\{(F_n)_E\}$ is said to be eventually contained in $F_E \in IFS(U)_E$ if and only if there is a positive integer m such that, $n \geq m$ implies $(F_n)_E \tilde{\subseteq} F_E$.

C. Convergence sequence

An IF-soft sequence $\{(F_n)_E\}$ in (U,τ,E) is said to be convergent and converge to $F_E \in IFS(U)_E$ if it is eventually contained in each neighbourhood of F_E and we say that the sequence $\{(F_n)_E\}$ has the limit F_E.

We write $\lim_{n\to\infty}(F_n)_E = \left(\lim_{n\to\infty} F_n\right)_E = F_E$

or $(F_n)_E \to F_E$ as n → ∞ or simply $F_n \to F$ as n → ∞.

Example 2

If we consider an IF-soft sequence $\{(F_n)_E\}$ as in example 1, then $\lim_{n\to\infty}(F_n)_E =$

$$\begin{aligned}\lim_{n\to\infty}\{ e_1 &= \{\langle h_1,([0.1,3/5n])\rangle,\langle h_2,([1/8n,0.3])\rangle,\\ &\quad \langle h_3,([1/5n,0.5])\rangle,\langle h_4,([0,2/5n])\rangle\}\\ e_2 &= \{\langle h_1,([0.1,0.3])\rangle,\langle h_2,([1/8n,3/5n])\rangle,\\ &\quad \langle h_3,([2/7n,3/5n])\rangle,\langle h_4,([0,0.5])\rangle\}\\ e_3 &= \{\langle h_1,([1/8n,0.5])\rangle,\langle h_2,([0.2,1/2n])\rangle,\\ &\quad \langle h_3,([0.1,2/5n])\rangle,\langle h_4,([1/5n,0.3])\rangle\}\end{aligned}$$

$$\begin{aligned}= \{ e_1 &= \{\langle h_1,([0.1,0])\rangle,\langle h_2,([0,0.3])\rangle,\\ &\quad \langle h_3,([0,0.5])\rangle,\langle h_4,([0,0])\rangle\}\\ e_2 &= \{\langle h_1,([0.1,0.3])\rangle,\langle h_2,([0,0])\rangle,\\ &\quad \langle h_3,([0,0])\rangle,\langle h_4,([0,0.5])\rangle\}\\ e_3 &= \{\langle h_1,([0,0.5])\rangle,\langle h_2,([0.2,0])\rangle,\\ &\quad \langle h_3,([0.1,0])\rangle,\langle h_4,([0,0.3])\rangle\}\end{aligned}$$

Proposition 1

If the neighbourhood system of each IF-soft set in (U,τ,E) is countable, then $F_E \in IFS(U)_E$ is open iff each IF-soft sequence $\{(F_n)_E\}$ which converges to $G_E \in IFS(U)_E$ contained in F_E is eventually contained in F_E.

Proof. Since F_E is an open in (U,τ,E), F_E is a neighbourhood of G_E. Hence, $\{(F_n)_E\}$ is eventually contained in F_E.

Conversely, for each $G_E \tilde{\subseteq} F_E$, let $(G_1)_E, (G_2)_E, \ldots, (G_n)_E, \ldots$ be the neighbourhood system of G_E and let $(H_n)_E = \tilde{\cap}_{i=1}^{n}(G_i)_E$, then $\{(H_n)_E\}$ is an IF-soft sequence, which is eventually contained in each neighbourhood of G_E. Hence, there is an integer m such that for $n \geq m$, $(H_n)_E \tilde{\subseteq} F_E$. Thus $(H_n)_E$ are neighborhood's of G_E. This implies F_E is a neighbourhood of G_E and hence F_E is open.

Proposition 2

If $F_E \in IFS(U)_E$ is open, then each IF-soft sequence $\{(F_n)_E\}$ which converges to $G_E \in IFS(U)_E$ contained in F_E is eventually contained in F_E.

Proof. Since F_E is open, F_E is a neighbourhood of G_E and since $\{(F_n)_E\}$ converges to G_E it is eventually contained in each neighbourhood of G_E. Hence, $\{(F_n)_E\}$ is eventually contained in F_E.

D. Subsequence

Let *f* be a mapping over the set of positive integers. Then the IF-soft sequence $\{(G_n)_E\}$ is a subsequence of a sequence $\{(F_n)_E\}$ iff there is a map *f* such that $(G_i)_E = (F_{f(i)})_E$ and for each integer m, there is an integer n_o such that $f(i) \geq m$ whenever $i \geq n_o$.

Example 3

Let $U = \{h_1, h_2, h_3, h_4\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$ and $A = \{e_1, e_2, e_3\}$.

If we chose for n=1, 2, 3….

$$(F_n)_E = \{e_1 = \{\langle h_1,([0.1,3/5n])\rangle,\langle h_2,([1/8n,0.3])\rangle,\ \langle h_3,([1/5n,0.5])\rangle,\langle h_4,([0,2/5n])\rangle\}$$
$$e_2 = \{\langle h_1,([0.1,0.3])\rangle,\langle h_2,([1/8n,3/5n])\rangle,\ \langle h_3,([2/7n,3/5n])\rangle,\langle h_4,([0,0.5])\rangle\}$$
$$e_3 = \{\langle h_1,([1/8n,0.5])\rangle,\langle h_2,([0.2,1/2n])\rangle,\ \langle h_3,([0.1,2/5n])\rangle,\langle h_4,([1/5n,0.3])\rangle\}$$

and

$$(G_n)_E = \{e_1 = \{\langle h_1,([0.1,3/4n])\rangle,\langle h_2,([1/8n,0.4])\rangle,\ \langle h_3,([1/7n,0.6])\rangle,\langle h_4,([0,2/5n])\rangle\}$$
$$e_2 = \{\langle h_1,([0.1,0.5])\rangle,\langle h_2,([1/8n,3/5n])\rangle,\ \langle h_3,([2/7n,3/5n])\rangle,\langle h_4,([0,0.6])\rangle\}$$
$$e_3 = \{\langle h_1,([1/8n,0.5])\rangle,\langle h_2,([0.2,1/2n])\rangle,\ \langle h_3,([0.1,2/5n])\rangle,\langle h_4,([1/5n,0.3])\rangle\}$$

Then $\{(G_n)_E\}$ is a subsequence of the IF-soft sequence $\{(F_n)_E\}$.

E. Complement of sequence

The complement of an IF-soft sequence $\{(F_n)_E\}$ in (U,τ,E) is denoted by $\{(F_n)_E\}^C$ and is defined by $\{(F_n)_E\}^C = \{(F_n)_E{}^C\} \ = \{(F_n^C)_E\}$

Example 4

If we consider an IF-soft sequence $\{(F_n)_E\}$ as in example 1, then the complement of $\{(F_n)_E\}$ is $\{(F_n)_E\}^C = \{(F_n)_E{}^C\} = \{(F_n{}^C)_E\}$

where for n=1, 2, 3,…….

$$(F_n^C,A) = \{e_1 = \{\langle h_1,([2/5n,0.3])\rangle,\langle h_2,([0.2,1/2n])\rangle,\ \langle h_3,([2/5n,0.3])\rangle,\langle h_4,([1/5n,1/2n])\rangle\}$$
$$e_2 = \{\langle h_1,([0.2,0.3])\rangle,\langle h_2,([2/5n,1/4n])\rangle,\ \langle h_3,([2/7n,0.3])\rangle,\langle h_4,([1/5n,1/8n])\rangle\}$$
$$e_3 = \{\langle h_1,([0.2,1/4n])\rangle,\langle h_2,([1/8n,0.3])\rangle,\ \langle h_3,([1/5n,0.3])\rangle,\langle h_4,([1/5n,3/5n])\rangle\}$$

F. Increasing sequence

An IF-soft sequence $\{(F_n)_E\}$ is said to be increasing sequence iff for each positive integer n, $(F_n)_E \tilde{\subseteq} (F_{n+1})_E$, i.e. $(F_1)_E \tilde{\subseteq} (F_2)_E \tilde{\subseteq} (F_3)_E \tilde{\subseteq} \ldots.$

G. Decreasing sequence

An IF-soft sequence $\{(F_n)_E\}$ is said to be decreasing sequence iff for each positive integer n, $(F_n)_E \tilde{\supseteq} (F_{n+1})_E$, i.e. $(F_1)_E \tilde{\supseteq} (F_2)_E \tilde{\supseteq} (F_3)_E \tilde{\supseteq} \ldots.$

H. Monotonic sequence

An IF-soft sequence $\{(F_n)_E\}$ is said to be monotonic if and only if the sequence is either increasing or decreasing sequence.

Example 5

Let $U=\{h_1,h_2,h_3,h_4\}$, $E=\{e_1,e_2,e_3,e_4,e_5\}$ and $A=\{e_1,e_2,e_3\}$.

If we chose for n=1, 2, 3….

$$(F_n)_E=\{e_1=\{\langle h_1,([0.1,0.5])\rangle,\langle h_2,([0.1,0.3])\rangle,\langle h_3,([0.2,0.5])\rangle,\langle h_4,([0,2/5n])\rangle\}$$
$$e_2=\{\langle h_1,([0.1,0.3])\rangle,\langle h_2,([0.1,1/7n])\rangle,\langle h_3,([0.2,3/5n])\rangle,\langle h_4,([0,0.5])\rangle\}$$
$$e_3=\{\langle h_1,([1/4-1/8n,0.5])\rangle,\langle h_2,([0.2,1/2n])\rangle,\langle h_3,([0,2/5n])\rangle,\langle h_4,([0,0.5])\rangle\}$$

and

$$(G_n)_E=\{e_1=\{\langle h_1,([1/5n,0.3])\rangle,\langle h_2,([0.2,1/2-1/2n])\rangle,\langle h_3,([1/5n,0.3])\rangle,\langle h_4,([0,1/2-1/2n])\rangle\}$$
$$e_2=\{\langle h_1,([0.1,0.4])\rangle,\langle h_2,([1/5n,1/2-1/4n])\rangle,\langle h_3,([0,0.3])\rangle,\langle h_4,([1/5n,1/4-1/8n])\rangle\}$$
$$e_3=\{\langle h_1,([0.2,0.4])\rangle,\langle h_2,([0,0.3])\rangle,\langle h_3,([1/5n,0.3])\rangle,\langle h_4,([2/5n,1/3-1/5n])\rangle\}$$

Then the IF-soft sequence $\{(F_n)_E\}$ is increasing sequence and the IF-soft sequence $\{(G_n)_E\}$ is decreasing sequence.

I. Frequently contained

An IF-soft sequence $\{(F_n)_E\}$ is said to be frequently contained in F_E iff for each positive integer n, there is a positive integer m such that, $n\geq m$ implies $(F_n)_E \tilde{\subseteq} F_E$.

J. Cluster of a sequence

$F_E\in IFS(U)_E$ in (U,τ,E) is said to be cluster of an IF-soft sequence $\{(F_n)_E\}$ if the sequence is frequently contained in every neighbourhood of F_E.

Proposition 3

If a neighbourhood system of each IF-soft set in (U,τ,E) is countable, then for F_E is a cluster of an IF-soft sequence $\{(F_n)_E\}$ there is a subsequence converging to F_E.

Proof. Let $(K_1)_E$, $(K_2)_E$,..., $(K_n)_E$,... be a neighbourhood system of F_E and let $(L_n)_E=\tilde{\cap}_{i=1}^{n}\{(K_i)_E\}$. Then $\{(L_n)_E : n=1,2,...\}$ is an IF-soft sequence such that $(L_{n+1})_E \tilde{\subseteq} (L_n)_E$ for each n and is eventually contained in each neighbourhood of F_E. For every positive integer *i*, choose *f(i)* such that $f(i)\geq i$ and $(F_{f(i)})_E \tilde{\subseteq} (L_i)_E$ and hence $\{(F_{f(i)})_E : i=1,2,...\}$ is a subsequence of the sequence $\{(F_n)_E : n=1,2,...\}$, which converges to F_E.

Proposition 4

Let F_E be a cluster of $\{(F_n)_E\}$ and F_E contained in G_E. If G_E is open, then the sequence is frequently contained in G_E.

Proof. Since G_E is open and hence G_E is a neighbourhood of F_E. Also, since F_E be a cluster of the sequence $\{(F_n)_E\}$ so by the definition of cluster, the sequence $\{(F_n)_E\}$ is frequently contained in every neighbourhood of F_E and hence, $\{(F_n)_E\}$ is frequently contained in G_E.

III. CONCLUSION

Fuzzy sets and soft sets are important mathematical tools for dealing with uncertainties and vagueness. In this research work, we have introduced a new type of IF-soft sequence in IF-soft topological spaces together with some basic properties, taking the whole parameter set. The concepts of subsequence, increasing sequence, decreasing sequence and convergence sequence of IF-soft sets are proposed. We also introduce the concepts of clusters of IF-soft sequences and study their basic properties.

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