International Journal of Advanced Research in Engineering and Applied Sciences ISSN: 2278-6252

STIFF FLUID LRS BIANCHI -I COSMOLOGICAL MODELS WITH VARYING G AND Λ

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Abstract: *I have discussed about a LRS Bianchi type-I cosmological model filled with stiff fluid, variable gravitational constant and cosmological constants. The cosmological models are obtained by assuming the cosmological term inversely proportional to scale factor. The physical significance of the cosmological models are also discussed.*

Key words:- *LRS Bianchi type-I,Variable cosmological term. Stiff fluid*

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1. INTRODUCTION

After the cosmological constant was first introduced into general relativity by Einstein, its significance was studied by various cosmologists (for example Petrosian, V1975), but no satisfactory results of its meaning have been reported as yet. Zel'dovich (1968) has tried to visualize the meaning of this term from the theory of elementary particles. Further, Linde (1974) has argued that the cosmological term arises from spontaneous symmetry breaking and suggested that the term is not a constant but a function of temperature. It is also well known that there is a certain degree of anisotropy in the actual universe. Therefore, we have chosen the metric for the LRS cosmological model to be Bianchi type-I.

Solutions to the field equations may also be generated by law of variation of scale factor which was proposed by Pavon, D. (1991). In earlier literature cosmological models with cosmological term is proportional to scale factor have been studied by Holy, F. et al(1997), Olson, T.S. et al. (1987), Pavon, D. (1991), Beesham, A (1994), Maia, M.D. et al. (1994), Silveria, V. et al. (1994,1997), Torres, L.F.B. et al. (1996). Chen and Wu (1990) considered Λ varying R^{-2} (R is the scale factor) Carvalho and Lima (1992) generated it by taking $\Lambda = \alpha R^{-2} + \beta H^2$ where R is the scale factor of Robertson-Walker metric, H is the Hubble parameter and α, β are adjustable dimensionless parameters .

The idea of variable gravitational constant G in the framework of general relativity was first proposed by Dirac (1937). Lau (1983) working in the framework of general relativity, proposed modification linking the variation of G with that of Λ. This modification allows us to use Einstein's field equations formally unchanged since variation in Λ is accompanied by a variation of G. A number of authors investigated Bianchies models, using this approach (Abdel-Rahman 1990; Berman 1991; Sisterio1991; Kalligas et al. 1992; Abdussattar and Vishwakarma 1997; Vishwakarma 2000,2005; Pradhan et al. 2006; Singh et al 2007; Singh and Tiwari 2007 Tiwari, R.K 2008,).

In this paper I have considered a LRS Bianchi type-I cosmological model with variables G and Λ filled with stiff fluid . We have obtained exact solutions of the field equations by assuming that cosmological term is inversely proportional to R (where R is scale factor). The paper is organized as follows. Basic equations of the models in sec. 2. and solution in sec. 3. The physical behavior of the model is discussed in detail is last section.

2. METRIC AND FILED EQUATIONS :

We consider spatially homogeneous and anisotropic LRS Bianchi type-I metric

$$ds^2 = -dt^2 + A^2(t)\, dx^2 + B^2(t)\, (dy^2 + dz^2) \qquad .....(1)$$

The energy-momentum tensor *Tij* for perfect fluid distribution is given by

$$T_{ij} = (\rho + p)\, v_i v_j + p g_{ij} \qquad .....(2)$$

where ρ is the energy density of the cosmic matter and p is its pressure, v_i is the four velocity vector such that $v_i v^i = 1$.

We take the equation of state (Zel'dovich 1962)

$$p = \rho \;, \qquad \omega = 1 \qquad .....(3)$$

The Einstein's field equations with time dependent G and Λ given by (Weinberg 1972)

$$R_{ij} - \tfrac{1}{2} R g_{ij} = -8\pi\, G(t)\, T_{ij} + \Lambda(t) g_{ij} \qquad .....(4)$$

For the metric (1) and energy - momentum tensor (2) in co-moving system of co-ordinates, the field equation (4) yields.

$$\frac{2\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 = -8\pi G p + \Lambda \qquad ....(5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G p + \Lambda \qquad ....(6)$$

$$\frac{2\dot{A}\dot{B}}{AB} + \left(\frac{\dot{B}}{B}\right)^2 = 8\pi G \rho + \Lambda \qquad ....(7)$$

In view of vanishing divergence of Einstein tensor, we have

$$8\pi G\left[\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}\right)\right]+8\pi\rho\dot{G}+\dot{\Lambda}=0 \qquad ....(8)$$

The usual energy conservation equation $T^{j}_{i;j}=0,$ yields

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}\right)=0 \qquad ....(9)$$

Equation (8) together with (9) puts G and Λ in some sort of coupled field given by

$$8\pi\rho\dot{G}+\dot{\Lambda}=0 \qquad ....(10)$$

Here and elsewhere a dot denotes for ordinary differentiation with respect to t. From (10) implying that Λ is a constant whenever G is constant.

Let R be the average scale factor of LRS Bianchi type -I universe i.e.

$$R^3 = AB^2 \qquad .....(11)$$

Using equation (3) in equation (9) and then integrating, we get,

$$.\ \rho=\frac{k}{R^6} \qquad .....(12)$$

where k > 0 is constant of integration.

From (5), (6) and (7), we obtain

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_1}{R^3} \qquad .....(13)$$

where k_1 is constant of integration. The Hubble parameter H, volume expansion θ, sheer σ and deceleration parameter q are given by

$$H=\frac{\theta}{3}=\frac{\dot{R}}{R}$$

$$\sigma=\frac{k}{\sqrt{3}R^3},$$

$$q=-1-\frac{\dot{H}}{H^2}=-\frac{-R\ddot{R}}{\dot{R}^2}$$

Equations (5)-(7) and (9) can be written in terms of H, σ and q as

$$H^2(2q-1)-\sigma^2=8\pi Gp-\Lambda \qquad .....(14)$$

$$3H^2 - \sigma^2 = 8\pi G\rho + \Lambda \qquad .....(15)$$

Overduin and Cooperstock (1998) define

$$\rho_c = \frac{3H^2}{8\pi G} \qquad .....(16)$$

$$\rho_v = \frac{\Lambda}{8\pi G} \qquad .....(17)$$

and $$\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G\rho}{3H^2} \qquad .....(18)$$

are respectively critical density, vacuum density and density parameter

$$\dot{\rho} + 3(\rho + p)\frac{\dot{R}}{R} = 0 \qquad .....(19)$$

From (15), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi G\rho}{\theta^2} - \frac{\Lambda}{\theta^2}$$

Therefore, $0 \le \frac{\sigma^2}{\theta^2} \le \frac{1}{3}$ and $0 \le \frac{8\pi G\rho}{\theta^2} \le \frac{1}{3}$ for $\Lambda \ge 0$

Thus, the presence of positive Λ puts restriction on the upper limit of anisotropy, where as a negative Λ contributes to the anisotropy.

From (14), and (15), we have

$$\frac{d\theta}{dt} = -12\pi Gp - \frac{\theta^2}{2} + \frac{3\Lambda}{2} - \frac{3\sigma^2}{2} = -12\pi G(\rho + p) - 3\sigma^2$$

Thus the universe will be in decelerating phase for negative Λ, and for positive Λ, universe will slows down the rate of decrease. Also $\dot{\sigma} = -\frac{3\sigma\dot{R}}{R}$ implying that σ decreases in an evolving universe and it is negligible for infinitely large value of R.

3. SOLUTION OF THE FIELD EQUATIONS -

The system of equations (3), (5)-(7), and (10), supply only five equations in six unknowns (A, B, ρ, p, G and Λ). One extra equation is needed to solve the system completely. Holy, F. et al (1997) considered $\Lambda \propto a^{-3}$ whereas $\Lambda \propto a^{-m}$ (a is scale factor and m is constant) considered by Olson, T.S. et al. (1987), Pavon, D. (1991), Maia, M.D. et al. (1994), Silveria, V. et. al. (1994, 1997) , Torres, LF. B. et al. (1996).

Thus we take the decaying vacuum energy density

$$\Lambda = \frac{a}{R} \qquad .....(20)$$

where a is positive constants. Using eq. (12) and (20) in eq. (10),

we get

$$G = \frac{a}{40\pi k}\frac{R^5}{} \qquad .....(21)$$

From eq (14), (15), (20) and (21) we get

$$\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 - \frac{a}{R} = 0 \qquad ....(22)$$

Integrating (22) we get

$$\frac{\dot{R}}{R} = H = \sqrt{\frac{2a}{5}}\left[\frac{1}{2}\sqrt{\left(\frac{2a}{5}\right)}\,t + t_0\right]^{-1} \qquad ...(23)$$

where the integration constant t_o is related to the choice of origin of time.

From (23) we obtain

$$R = \left[\frac{1}{2}\sqrt{\left(\frac{2a}{5}\right)}t + t_0\right]^2 \qquad .....(24)$$

By using (24) in (13) , the metric (1) assumes the form

$$ds^2 = -dt^2 + \left(\frac{1}{2}\sqrt{\frac{2a}{5}}\,t + t_0\right)^4 \times$$

$$\left[m_1^2\ \exp\left\{\frac{8k_1}{3}\sqrt{\frac{5}{2a}}\frac{1}{-5}\left(\frac{1}{2}\sqrt{\frac{2a}{5}}t + t_0\right)^{-5}\right\}dx^2 \right. \qquad .....(25)$$

$$\left. + m_2^2\ \exp\left\{\frac{-4k_1}{3}2\sqrt{\frac{5}{2a}}\frac{1}{-5}\left(\frac{1}{2}\sqrt{\frac{2a}{5}}t + t_0\right)^{-5}\right\}(dy^2 + dz^2)\right]$$

where m_1, m_2 are constants.

For the model (25), spatial volume V, matter density ρ, pressure p, gravitational constant G, cosmological constant Λ are given by

$$V = \left[\frac{1}{2}\sqrt{\left(\frac{2a}{5}\right)}t + t_0\right]^6 \quad .....(26)$$

$$\rho = p = \frac{k}{\left(\frac{1}{2}\sqrt{\frac{2a}{5}}t + t_0\right)^{12}} \quad .....(27)$$

$$G = \frac{a}{40\pi k}\left[\frac{1}{2}\sqrt{\frac{2a}{5}}t + t_0\right]^{10} \quad ....(28)$$

$$\Lambda = \frac{a}{\left[\frac{1}{2}\sqrt{\frac{2a}{5}}t + t_0\right]^2} \quad .....(29)$$

Expansion scalar θ and shear σ are given by

$$\theta = 3\sqrt{\frac{2a}{5}}\left[\frac{1}{2}\sqrt{\frac{2a}{5}}t + t_0\right]^{-1} \quad .....(30)$$

$$\sigma = \frac{k}{\sqrt{3}}\left[\frac{1}{2}\sqrt{\frac{2a}{5}}t + to\right]^{-6} \quad ....(31)$$

The density parameter is given by

$$\Omega = \frac{\rho}{\rho_c} = \frac{1}{6} \quad .....(32)$$

The deceleration parameter q for the model is

$$q = -\frac{1}{2} \quad ..(33)$$

In the model, we observe that , the spatial volume V is zero at t = $\frac{-t_0}{\frac{1}{2}\sqrt{\frac{2a}{5}}} = t''$ and expansion scalar θ is infinite at t= t" which shows that the universe starts evolving with zero volume and infinite rate of expansion at t= t". Initially at t = t" the energy density ρ, pressure p, Λ and shear scalar σ are infinite. As t increases the spatial volume increases but the expansion scalar decreases. Thus the expansion rate decreases as time increases. As t tends to ∞ the spatial volume V becomes infinitely large. As t increases all the parameters p, ρ, Λ, θ, ρ_c, ρ_v, decrease and tend to zero asymptotically. Therefore, the model essentially gives an empty universe for large t. The ratio $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$, which shows that model approaches isotropy for large values of t. The gravitational constant G(t) is zero at t =t" and as t increases, G increases and it becomes infinite large at late times.

Further, we observe that $\Lambda \alpha \frac{1}{t^2}$ which follows from the model of Kalligas et al. (1996); Berman (1990); Berman and Som (1990); Berman et al. (1989) and Bertolami (1986a, b). This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe.

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