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# BIANCHI TYPE-I COSMOLOGICAL MODELS WITH DUST FLUID IN GENERAL RELATIVITY

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**Abstract:** *In this paper, we have investigated Bianchi type -I cosmological model with dust fluid. The cosmological models are obtained by assuming the cosmological term inversely proportional to R (R is scale factor ).Some physical properties of the cosmological models are also discussed.*

***Keywords***:- *Bianchi-I cosmological model Variable cosmological term. Dust fluid .*

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## 1. INTRODUCTION :

The simplest homogeneous and anisotropic models are Bianchi type-I whose optical sections are that but the expansion or contraction rate is directional dependent. The advantages of these anisotropic models are that they have a significant role in the description of the evolution of the early phase of the universe and they help in finding move general cosmological models than the isotropic Friedman- Robertson Walker models. The isotropy of the present day universe makes the Bianchi-I model a prime for studying the possible effects of an anisotropy in the early universe on modern day data observations.

Solutions to the field equations may also be generated by law of variation of scale factor which was proposed by Pavon, D. (1991). The behavior of the cosmological scale factor R (t) in solution of Einstein's field equations with Robertson-Walker line elements has been the subject of numerous studies. In earlier literature cosmological models with cosmological term is proportional to scale factor have been studied by Holy, F. et al(1997), Olson, T.S. et al. (1987), Beesham, A (1994), Maia, M.D. et al. (1994), Silveria, V. et al. (1994,1997), Torres, L.F.B. et al. (1996). Chen and Wu (1990) considered $\Lambda$ varying $R^{-2}$ (R is the scale factor) Carvalho and Lima (1992) generated it by taking $\Lambda = \alpha R^{-2} + \beta H^2$ where R is the scale factor of Robertson-Walker metric, H is the Hubble parameter and $\alpha$, $\beta$ are adjustable dimensionless parameters on the basis of quantum field estimations in the curved expanding background.

The idea of variable gravitational constant G in the framework of general relativity was first proposed by Dirac (1937). Lau (1983) working in the framework of general relativity, proposed modification linking the variation of G with that of $\Lambda$. . A number of authors investigated Bianchies models, using this approach (Abdel-Rahman 1990; Berman 1991; Kalligas et al. 1992; Abdussattar and Vishwakarma 1997; Vishwakarma 2005; Pradhan et al. 2006; Singh and Tiwari 2007). Borges and Carneiro (2005), Singh et al. (2007) have considered as cosmological term is proportional to the Hubble parameter in FRW model and Bianchi type-I model with variable G and $\Lambda$.

In this paper we have investigated homogeneous Bianchi type -I space time with variables G and $\Lambda$ containing matter in the form of dust fluid. We have obtained exact solutions of the field equations by assuming that cosmological term is inversely proportional to R.. The paper is organized as follows. Basic equations of the models in sec. 2. and solution in sec. 3. The physical behavior of the model is discussed in detail is last section.

## 2. METRIC AND FILED EQUATIONS :

We consider the space-time admitting Bianchi type-I group of motion in the form

$ds^2 = -dt^2 + A^2(t)\, dx^2 + B^2(t)\, dy^2 + C^2(t)\, dz^2$ .....(1)

We assume that the cosmic matter is represented by the energy momentum tensor of a perfect fluid

$T_{ij} = (\rho + p)\, v_i v_j + p g_{ij}$ .....(2)

where $\rho$ is the energy density of the cosmic matter and p is its pressure, $v_i$ is the four velocity vector such that $v_i v^i = 1$.

We take the equation of state

$p = \omega\rho$ , $0 \leq \omega \leq 1$ ....(3)

Here we take $\omega=0$, then $p=0$.

The Einstein's field equations with time dependent G and $\Lambda$ given by (Weinberg 1972)

$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi\, G(t)\, T_{ij} + \Lambda(t) g_{ij}$ .....(4)

For the metric (1) and energy - momentum tensor (2) in co-moving system of co-ordinates, the field equation (4) yields.

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G p + \Lambda \quad ....(5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -8\pi G p + \Lambda \quad ....(6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G p + \Lambda \quad ....(7)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = 8\pi G \rho + \Lambda \quad ....(8)$$

In view of vanishing divergence of Einstein tensor, we have

$$8\pi G\left[\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right] + 8\pi\rho\dot{G} + \dot{\Lambda} = 0 \quad ....(9)$$

The usual energy conservation equation $T^{j}_{i;j} = 0,$ yields

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \quad ....(10)$$

Equation (9) together with (10) puts G and $\Lambda$ in some sort of coupled field given by

$$8\pi\rho\dot{G}+\dot{\Lambda}=0 \quad ....(11)$$

Here and elsewhere a dot denotes for ordinary differentiation with respect to t. From (11) implying that $\Lambda$ is a constant whenever G is constant. Using equation (3) in equation (10) and then integrating, we get,

$$\rho=\frac{k}{R^3} \quad .....(12)$$

where k > 0 is constant of integration.

Let R be the average scale factor of Bianchi type -I universe i.e.

$$R^3 = ABC \quad .....(13)$$

From (5), (6) and (7), we obtain

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_1}{R^3} \quad .....(14)$$

and

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{k_2}{R^3} \quad .....(15)$$

where $k_1$ and $k_2$ are constant of integration. The Hubble parameter H, volume expansion $\theta$, sheer $\sigma$ and deceleration parameter q are given by

$$H=\frac{\theta}{3}=\frac{\dot{R}}{R}$$

$$\sigma=\frac{k}{\sqrt{3}R^3},$$

$$q=-1-\frac{\dot{H}}{H^2}=-\frac{-R\ddot{R}}{\dot{R}^2}$$

Equations (5)-(8) and (10) can be written in terms of H, $\sigma$ and q as

$$H^2(2q-1)-\sigma^2=8\pi Gp-\Lambda \quad .....(16)$$

$$3H^2-\sigma^2=8\pi G\rho+\Lambda \quad .....(17)$$

$$\dot{\rho}+3(\rho+p)\frac{\dot{R}}{R}=0 \quad .....(18)$$

From (16), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi G\rho}{\theta^2} - \frac{\Lambda}{\theta^2}$$

Therefore, $0 \le \frac{\sigma^2}{\theta^2} \le \frac{1}{3}$ and $0 \le \frac{8\pi G\rho}{\theta^2} \le \frac{1}{3}$ for $\Lambda \ge 0$

Thus, the presence of positive $\Lambda$ puts restriction on the upper limit of anisotropy, where as a negative $\Lambda$ contributes to the anisotropy.

From (16), and (17), we have

$$\frac{d\theta}{dt} = -12\pi Gp - \frac{\theta^2}{2} + \frac{3\Lambda}{2} = 12\pi G(\rho + p) - 3\sigma^2$$

Thus the universe will be in decelerating phase for negative $\Lambda$, and for positive $\Lambda$, universe will slows down the rate of decrease. Also $\dot{\sigma} = -\frac{3\sigma\dot{R}}{R}$ implying that $\sigma$ decreases in an evolving universe and it is negligible for infinitely large value of R.

## 3. SOLUTION OF THE FIELD EQUATIONS -

The system of equations (3), (5)-(8), and (11), supply only six equations in seven unknowns (A,B,C, $\rho$, p, G and $\Lambda$). One extra equation is needed to solve the system completely. The $\Lambda \alpha$ $a^{-m}$ (a is scale factor and m is constant) considered by Olson, T.S. et al. (1987), Pavon, D. (1991), Maia, M.D. et al. (1994), Silveria, V. et. al. (1994, 1997) , Torres, LF. B. et al. (1996).

Thus we take the decaying vacuum energy density

$$\Lambda = \frac{c}{R} \qquad .....(19)$$

where c is positive constants. Using eq. (12) and (19) in eq. (11),

we get

$$G = \frac{c}{16\pi k}\frac{R^2}{} \qquad .....(20)$$

From eq (16), (17), (19) and (20) we get

$$\frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}}{R}\right)^2 - \frac{5c}{4R} = 0 \qquad .....(21)$$

Determining the time evolution of Hubble parameter, integrating (21), we get

$$\frac{\dot{R}}{R} = H = \sqrt{\frac{c}{2}}\left[\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)}\ t + t_0\right]^{-1} \qquad ....(22)$$

where $t_o$ is a constant of integration, The integration constant is related to the choice of origin of time.

From eq. (22), we obtain the scale factor

$$R = \left(\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)}\ \mathrm{t} + t_0\right)^2 \qquad ...(23)$$

By using eq (23) in (14) and (15), the metric (1), assumes the form

$$\begin{aligned} ds^2 = {} & -dt^2 + \left(\frac{1}{2}\sqrt{\frac{c}{2}}t + t_0\right)^4 \times \\ & \left[m_1^2\ \exp 2\left\{\frac{(2k_1 + k_2)}{3}2\sqrt{\frac{2}{c}}\frac{1}{-5}\left(\frac{1}{2}\sqrt{\frac{c}{2}}t + t_0\right)^{-5}\right\}dx^2\right. \\ & + m_2^2\ \exp 2\left\{\frac{(k_2 - k_1)}{3}2\sqrt{\frac{2}{c}}\frac{1}{-5}\left(\frac{1}{2}\sqrt{\frac{c}{2}}t + t_0\right)^{-5}\right\}dy^2 \\ & \left. + m_3^2\ \exp 2\left\{\frac{-(k_1 + 2k_2)}{3}2\sqrt{\frac{2}{c}}\frac{1}{-5}\left(\frac{1}{2}\sqrt{\frac{c}{2}}t + t_0\right)^{-5}\right\}dz^2\right] \end{aligned} \qquad ....(24)$$

where $m_1$, $m_2$ and $m_3$ are constants.

For the model (24), the spatial volume V, matter density $\rho$, pressure p gravitational constant G, cosmological constant $\Lambda$ are given by

$$V = \left(\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)}\ \mathrm{t} + t_0\right)^6 \qquad .....(25)$$

$$\rho = \frac{k}{\left[\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)}\ t + \mathrm{to}\right]^6} \qquad .....(26)$$

$$p = 0 \qquad ......(27)$$

$$G = \frac{c}{16\pi k}\left(\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)}\ \mathrm{t} + t_0\right)^4 \qquad .....(28)$$

$$\Lambda = c\left[\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)}\; t + t_0\right]^{-2} \quad ....(29)$$

Expansion scalar $\theta$ and shear $\sigma$ are given by

$$\theta = 3\sqrt{\frac{c}{2}}\;\left[\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)}\,t + to\right]^{-1} \quad ....(30)$$

$$\sigma = \frac{k}{\sqrt{3}}\left[\frac{1}{2}\sqrt{\left(\frac{c}{2}\right)}\,t + to\right]^{-6} \quad .....(31)$$

The deceleration parameter q for the model is

$$q = -\frac{1}{2} \quad ....(32)$$

Thus for the model (24), the spatial volume V is zero at t=t' where t' = $\dfrac{-t_0}{\frac{1}{2}\sqrt{\frac{c}{2}}}$ and expansion scalar $\theta$ is infinite, which shows that the universe starts evolving with zero volume at t=t' with an infinite rate of expansion. The scale factors also vanish at t=t' and hence the space-time exhibits point type singularity at initial epoch. The energy density, shear scalar diverges at the initial singularity. As t increases the scale factors and spatial volume increase but the expansion scalar decreases. Thus the rate of expansion slows down with increase in time. Also $\rho$, $\sigma$, $\rho_v$, $\rho_c$, $\Lambda$ decrease as t increases. As t $\rightarrow\infty$ scale factors and volume become infinite whereas $\rho$, $\sigma$, $\rho_v$, $\rho_c$, and $\Lambda$ tend to zero. Therefore, the model would essentially give an empty universe for large time t. Gravitational constant G(t) is zero at t =t' and as t increases G(t) also increases. The cosmological constant $\Lambda$(t) $\alpha$ $1/t^2$ which follows from the model of Kalligas et al. (1996); Berman (1990); Berman and Som (1990); Berman et al. (1989) . This form of $\Lambda$ is physically reasonable as observations suggest that $\Lambda$ is very small in the present universe.

The ratio $\frac{\sigma}{\theta} \rightarrow 0$ as $t \rightarrow \infty$ . So the model approaches isotropy for large value of t. Thus the model represents shearing, non-rotating and expanding model of the universe with a big bang start approaching to isotropy at late times.Finally, the solutions presented in the paper are new and useful or better understanding of the evolution of the universe in Bianchi type-I space-time with variable G and $\Lambda$.

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