FINITE DIFFERENCE ANALYSIS OF A FREE CONVECTION FLOW ALONG A VERTICAL WALL IN A POROUS MEDIUM

T. Gopal Reddy*

M. Rama Chandra Reddy**

B. Rama Bhupal Reddy***

Abstract: *In this paper studied on free convection flow along a vertical wall in a porous medium. The flow is in two dimensional. It is well known that for constant wall temperature and permeability, the problem has no similarity solution and hence we have to deal with governing partial differential equations of the system. The steady state equations are solved with the help of highly implicit difference scheme. The difference equations are non-linear which have been solved by using the finite difference technique. Velocity and temperature curves are presented from the results of computational work.*

Keywords: *Free convection, vertical wall, Porous Medium*

*Jr. Lecturer in Mathematics, Government Junior College, Kurnool, A.P., India.

**Reader in Mathematics, S.K.S.C. Degree College, Proddatur, A.P., India.

***Associate Professor in Mathematics, K.S.R.M. College of Engineering, Kadapa, A.P., India.

1. INTRODUCTION

Flow through and past porous media has attracted considerable research activities in recent years because of its several important applications notably in the flow of oil through porous rocks, extractions of energy from the geothermal regions, the evaluation of the capability of heat removal from particulate nuclear fuel debris that may result from hypothetical accident in a nuclear reactor, the filtration of solids from liquids, flow of liquids through ion-exchange beds, drug permeation through human skin, chemical reactors for economical separation or purification through human skin, chemical reactors for economical separation or purification of mixtures, the study of transfer of nutrients from synovial fluid to cartilages in synovial joints the study of dispersion of cholesterol and other fat substances from arteries to endothelium and so on..

Free convective flow past vertical plate has been studied extensively by Ostrach [22, 23] and many others. Siegel [28] investigated the transient free convection from a vertical flat plate. The free convective heat transfer on a vertical semi-infinite plate has been investigated by Berezovsky et al. [4]. Martynenko et al. [19] investigated the laminar free convection from a vertical plate. In all these papers, the plate was assumed to be maintained at a constant temperature, which is also the temperature of the surrounding stationary fluid. But in industrial applications, quite often the plate temperature starts oscillating about a non-zero mean temperature. In many engineering applications, transient free convection flow occurs as such a flow acts as a cooling device. However, free-convection flow is enhanced by superimposing oscillating temperature on the mean plate temperature. Again transient natural convection is of interest in the early stage of melting adjacent to heated surface or in transient heating of insulating air gaps by heat input at the start-up of furnaces. Soundalgekar [32] studied the unsteady free convection flow past an infinite vertical plate with constant suction and mass transfer. It was assumed that the plate temperature oscillates in such a way that its amplitude is small.

The introduction of the boundary layer approximation in the porous media literature has intensified research efforts on the analytical studies of convective heat transfer about heated surfaces in porous media Cheng [10]. One of the most basic problems in natural convection heat transfer is buoyant boundary layer flow along a heated vertical wall adjacent to a fluid-saturated porous medium. This basic problems was analyzed for the first time by Cheng and Minkowycz [11] who relies on the theoretical frame work provided by boundary layer theory. The same phenomenon has been studied in related configurations such as the flow on the out side of a vertical cylindrical surface imbedded a porous medium Minkowycz, and Cheng [20], or the flow along the inner wall of cylindrical well filled with porous material Bejan [2]. The time-dependent boundary layer flow triggered by the sudden imposition of a temperature difference between a wall and fluid-saturated medium was studied by Ingham et.al. [13].

Among the practical considerations that stimulate the continuing interest in this flow is the engineering of efficient thermal insulation systems. This is why the vertical boundary layer concept has been used consistently in the analytical treatment of natural convection in an enclosed space filled with porous medium. The boundary layer treatment of enclosed porous layers begin with Weber's paper [36] and continued with the works of Simpkins and Blythe [29, 30], Blythe et.al. [6], Bejan [3] and Bergholz [5]. The Darcy model (which assumes proportionally between the velocity and pressure gradient has been extensively used to investigate a number of interesting fluid and heat transfer problems associated with heated bodies embedded in fluid-saturated porous media (e.g. Wooding [37]; Cheng [10], Cheng and Minkowycz [11]). The model, however is valid only for slow flows through porous media with low permeability. Muskat [21] added a velocity square term (known as the Forceimer term) to account for the porous inertia effect on the pressure drop, while Brinkman [7] introduced a viscous diffusion term to consider the boundary frictional drag on impermeable walls.

Kimura and Bejan [17] have analyzed natural convection in a stably heated corner where Darcy flow is driven by competing thermal gradients. Punyatmasing et al. [26] have analyzed the free convection boundary layer flow along a vertical surface in a porous medium by employing a generalized equation of Darcy's law in which the convection term is taken into account. The temperature of the wall and the permeability of the porous media is constant. Recently, the problems related to double-diffusive flow which combine heat and mass transfer have been addressed by Bejan and Khair [1] and Kumari et.al. [18]. Prasad et.al. [25] have experimentally studied convection in a vertical porous annulus comprising various combinations of ball sizes and fluids.

Problems involving vertical porous layers bounded by parallel wall have been extensively studied and a detailed review has been written by Cheng [10]. Cheng and Minkowycz and their associates [11, 12, 14, 15] have studied the free convection boundary layer flows by using Darcy's law as the momentum equation. Plumb and Huenefeld [24] have analyzed the buoyancy induced boundary layer adjacent to a vertical heated surface using a non-Darcy flow model. Darcy's law is considered to be valid for low speed flows, whereas the speed in the filter is not always small and the convective force may be important Yamamoto and Iwamura [38]. The effects of Mass Transform on the flow past a uniformly accelerated vertical plate which is either at uniform temperature or its supplied heat at constant rate is studied by Soundalgekar [31]. Treating a fluid-saturated porous medium as a continuum, Vafai and Tien [35] integrated the momentum equation or a fluid over a local control volume, and derived a volume averaged momentum equation. This generalized momentum equation for non-Darcy flows reveal the importance of the convective inertia term for highly porous material. Subsequently, Chen et.al. [9] evoked the boundary layer approximations and solved the generalized equation to investigate free convection from a vertical flat plate in highly porous medium, while the corresponding forced convection problems was treated by Kaviany [16], who employed the finite difference calculation procedure developed by Cebeci and Bradshaw [8], Kaviany also employed the Kaman-Pohlhansen integral relation and obtained useful asymptotic expressions for the local Nusselt number. He, however, dropped the Forchheimer term in his finite difference calculations.

Laminar free convection through a channel in the absence of porous material has been studied extensively by many authors like Sparrow et.al. [33], Ostrach [23], but the same in the presence of a porous medium has not been given much attention. The fully developed free convection in a viscous fluid flow through a vertical stratum was investigated considering the Brinkman model [7]. Effects of Free convection and Mass transform flow through a porous medium are studied by Rapid et al. [27], Tong and Subrahmanian [34] studied the effect of the no slip boundary condition on flows in two dimensional rectangular enclosures with R sufficiently high to allow exhibition of boundary layer characteristics. By using modified Oseen method [33] the boundary layer equations were solved which were derived from the Brikman's extended model [7] were solved by comparing the results to those obtained from the Darcy model.

2 NOMENCLATURE

C_P	Specific heat
g	Acceleration due to gravity
k_1	Permeability parameter
L	Characteristic length in X-direction
P	Dimensionless pressure difference
Pr	Prandtl number
T	Temperature
T_W	Wall temperature
T_∞	The free – stream temperature
u	The velocity in the x-direction
v	Velocity in the y-direction
U	Dimensionless velocity in the X-direction
V	Dimensionless velocity in the Y-direction
x,y	Co-ordinate axis
X,Y	Non-dimensional axis
ρ	Mass density
κ	Thermal Conductivity
μ	Dynamic viscosity
θ	Dimensionless Temperature

3 FORMULATION OF THE PROBLEM

A two dimensional free convection flow and heat transfer along a vertical wall in a porous media. The wall is maintained at different temperatures. The horizontal walls are adiabatic. The governing equations for the steady, viscous incompressible flow of an electrically conditioning fluid for the Brinkman-extended Darcy model are:

$$\rho \frac{D\bar{q}}{Dt} = -\nabla p^1 + \mu \nabla^2 \bar{q} + \rho g - \frac{\mu \bar{q}}{k_1} \tag{1}$$

$$\nabla \bullet \bar{q} = 0 \tag{2}$$

$$\rho\, C_p \left[\bar{q} \cdot \nabla \right] T = \kappa \nabla^2 T + \phi \tag{3}$$

Where ϕ = Dissipation function

$$= \mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2 + 2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right]$$

Where $\bar{q} = (u,v,0)$

The following assumptions are made:

(i) The flow is only in x and y direction

(ii) The energy dissipation is neglected

(iii) Pressure term will be neglected

(iv) The Boussinesq approximation has been used for the buoyancy term and constant properties have been assumed.

Introducing the above assumptions, the equations (1) to (3) for the Brinkman extended Darcy model are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (4)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=V\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{vu}{k_1} \qquad (5)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \qquad (6)$$

The boundary conditions are

At x = 0 : $u = 0, T = T_\infty$

At y = 0 : $u = 0, v = 0, T = T_w$ (7)

At $y \to \infty$: $u = 0, T = T_\infty$

The equations (4) to (6) and boundary conditions (7) are put in dimensionless from by using the following transformations:

$$U=\frac{vu}{\left(L^2 g\beta(T_w-T_\infty)\right)} \qquad K=\frac{k_1}{L^2} \qquad Y=\frac{y}{L} \qquad V=\frac{vL}{v} \qquad X=\frac{V^2x}{(g\beta(T_w-T_\infty))L^4}$$

$$\theta=\frac{(T-T_\infty)}{(T_w-T_\infty)} \qquad (8)$$

The non-dimensional form the governing equations are:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (9)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}-\frac{U}{K}+\theta \qquad (10)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \qquad (11)$$

The boundary conditions are

U(X, 0) = U(X, ∞) = U(0, Y) = V(X, 0) = 0

θ(X, 0) = 1, θ(X, ∞) = θ(0, Y) = 0 (12)

A variable mesh net work is introduced across the range of the problem as shown in figure 1. The finite difference approximation to the derivatives (9) – (11) as shown as follows:

Continuity equation

$$\frac{\partial V}{\partial Y}=\frac{V(j+1,k+1)-V(j+1,k)}{\Delta Y}$$

$$\frac{\partial U}{\partial X}=\frac{U(j+1,k+1)+U(j+1,k)-U(j,k+1)-U(j,k)}{2\Delta X}$$

Momentum equation:

$$\frac{\partial U}{\partial X}=\frac{U(j+1,k)-U(j,k)}{\Delta X}$$

$$\frac{\partial U}{\partial Y}=\frac{U(j+1,k+1)-U(j+1,k-1)}{2\Delta Y}$$

$$\frac{\partial^2 U}{\partial Y^2}=\frac{U(j+1,k+1)-2U(j+1,k)+U(j+1,k-1)}{(\Delta Y)^2}$$

$$\frac{dP}{dX}=\frac{P(j+1)-P(j)}{\Delta X}$$

Energy equation

$$\frac{\partial \theta}{\partial X}=\frac{\theta(j+1,k)-\theta(j,k)}{\Delta X}$$

$$\frac{\partial \theta}{\partial Y}=\frac{\theta(j+1,k+1)-\theta(j+1,k-1)}{2\Delta Y}$$

$$\frac{\partial^2 \theta}{\partial Y^2}=\frac{\theta(j+1,k+1)-2\theta(j+1,k)+\theta(j+1,k-1)}{(\Delta Y)^2}$$

4 FINITE DIFFERENCE SOLUTION

Writing equations (10), (9) and (11) in finite difference from and applying them to the (i, j) mesh point of a rectangular grid figure 1 as shown:

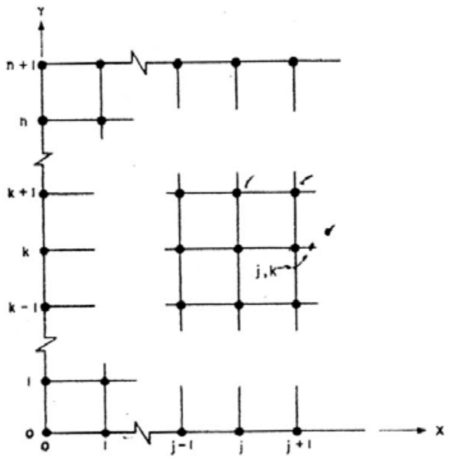

Figure 1 : Mesh scheme

$$U(i,j)\frac{U(i+1,j)-U(i,j)}{\Delta X}+V(i,j)\frac{U(i+1,j+1)-U(i+1,j-1)}{2\Delta Y}$$

$$=\frac{U(i+1,j+1)-2U(i+1,j)+U(i+1,j-1)}{(\Delta Y)^2}-\frac{U(i+1,j)}{K}+\theta(i+1,j) \tag{13}$$

$$\frac{U(j+1,k+1)+U(j+1,k)-U(j,k+1)-U(j,k)}{2\Delta X}$$

$$+\frac{V(i+1,j+1)-V(i+1,j)}{\Delta Y}=0 \tag{14}$$

$$U(i,j)\frac{\theta(i+1,j)-\theta(i,j)}{\Delta X}+V(i,j)\frac{\theta(i+1,j+1)-\theta(i+1,j-1)}{2\Delta Y}$$

$$=\frac{1}{\Pr}\frac{\theta(i+1,j+1)-2\theta(i+1,j)+\theta(i+1,j-1)}{(\Delta Y)^2} \tag{15}$$

The difference form selected here is highly implicit, that is, not only all Y-direction derivatives are evaluated at i+1 but the coefficient of non-linear convective terms are also evaluate at i+1. This representation is necessary since the usual implicit scheme in which the coefficients are evaluated at i is inconsistent for these conditions.

The difference equations (13) to (15) now, become

$$\left[\frac{-1}{(\Delta Y)^2}-\frac{V(i,j)}{2\Delta Y}\right]U(i+1,j-1)+\left[\frac{2}{(\Delta Y)^2}+\frac{U(i,j)}{\Delta X}+\frac{1}{K}\right]U(i+1,j)$$

$$+\left[\frac{-1}{(\Delta Y)^2}+\frac{V(i,j)}{2\Delta Y}\right]U(i+1,j+1)-\theta(i+1,j)=\frac{U(i,j)U(i,j)}{\Delta X} \tag{16}$$

$$V(i+1,j+1)=V(i+1,j)$$

$$-\Delta Y\frac{U(j+1,k+1)+U(j+1,k)-U(j,k+1)-U(j,k)}{2\Delta X} \tag{17}$$

$$\left[\frac{-1}{\Pr(\Delta Y)^2}-\frac{V(i+1,j)}{2\Delta Y}\right]\theta(i+1,j-1)+\left[\frac{2}{\Pr(\Delta Y)^2}+\frac{U(i+1,j)}{\Delta X}\right]\theta(i+1,j)$$

$$+\left[\frac{-1}{\Pr(\Delta Y)^2}+\frac{V(i+1,j)}{2\Delta Y}\right]\theta(i+1,j+1)=\frac{U(i+1,j)\theta(i,j)}{\Delta X} \tag{18}$$

Equations (16) to (18) together with the boundary conditions (12) are solved by Gauss elimination method which consists of solving the set of equations (16), (17) and (18) in that order repeatedly.

5 RESULTS AND DISCUSSION

The numerical solution of the equation is obtained first selecting the parameters that are involved such as K and Pr. The velocity and temperature distribution are analyzed for different sets of governing parameters K and Pr. We choose for P_r = 0.71 and K = 0, 2, 4 and 8. The corresponding profiles are platted in figures 2 to 17.

Velocity profiles are shown as a function of Y for X = 0.03, 0.05, 0.07, 0.09. Figures 3 to 5 shows that the variations of velocity for fixed Y for different values of K $\neq$ 0 and it is observed that velocity is increases significantly as the value of K increases. Figure 2 shows that the variation of velocity for fixed value of 1/K = 0 for different values of Y. Velocity profiles are shown as a function of X for Y=1, 2, 3 and 4. Figures 7 to 9 shows that the variations of velocity for fixed X for different values of K $\neq$ 0 and it is observed that velocity is increases significantly as the value of K increases. Figure 6 shows that the variation of velocity for fixed value of 1/K = 0 for different values of X.

Temperature profiles are shown as a function of Y for X = 0.03, 0.05, 0.07, 0.09. Figures 11 to 13 shows that the variations of temperature for fixed value of Y for different values of K $\neq$ 0 and it is observed that the temperature increases for K increases. Figure 10 shows that the variation of temperature for fixed value of 1/K = 0 for different values of Y. Temperature profiles are shown as a function of X for Y=1, 2, 3 and 4. Figures 15 to 17 shows that the variations of velocity for fixed X for different values of K $\neq$ 0 and it is observed that velocity is increases significantly as the value of K increases. Figure 14 shows that the variation of temperature for fixed value of 1/K = 0 for different values of X.

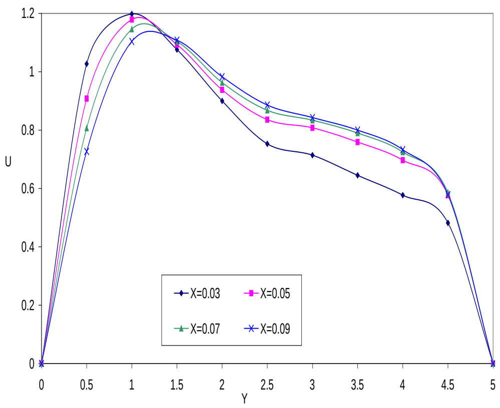

Figure 2: Velocity profile for X and K = 0

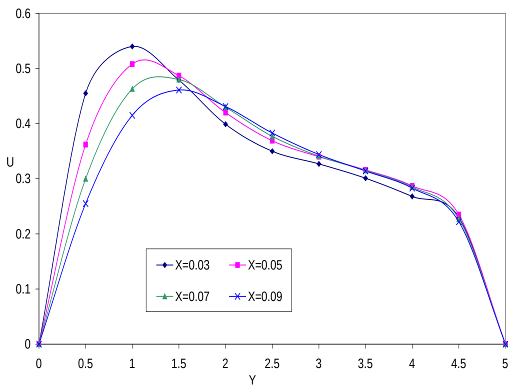

Figure 3: Velocity profile for X and K = 2

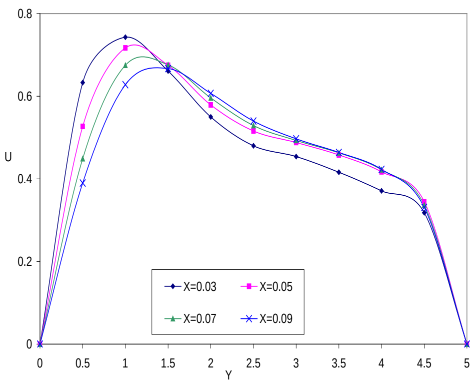

Figure 4: Velocity profile for X and K = 4

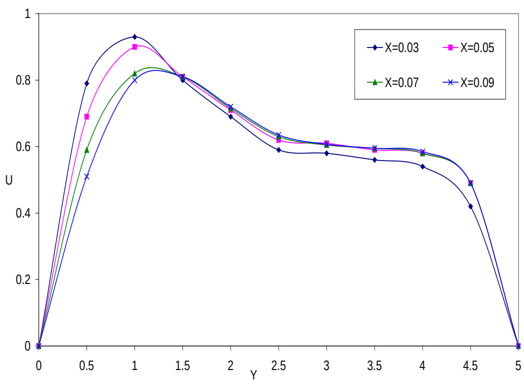

Figure 5: Velocity profile for X and K = 8

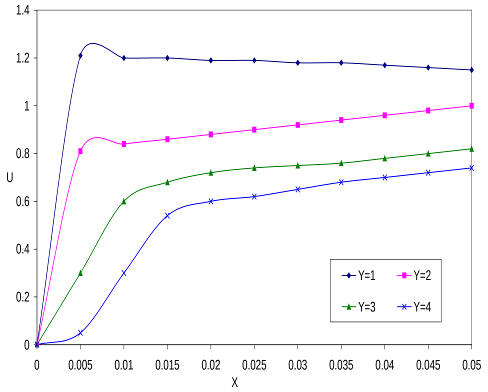

Figure 6: Velocity profile for Y and K = 0

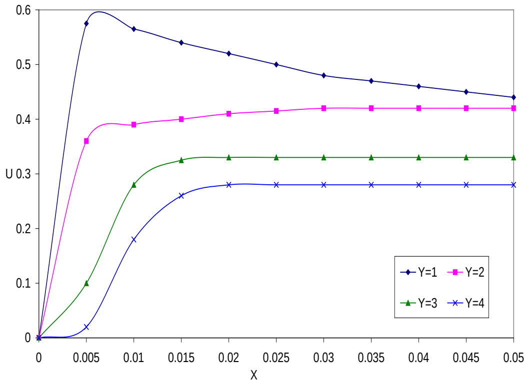

Figure 7: Velocity profile for Y and K = 2

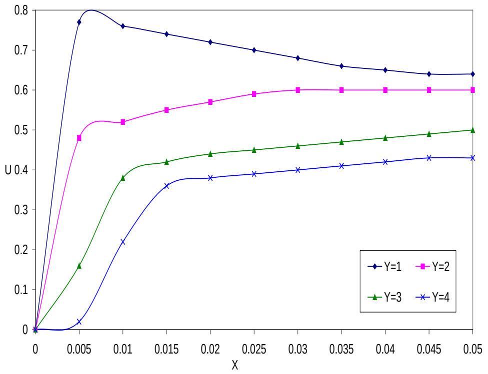

Figure 8: Velocity profile for Y and K = 4

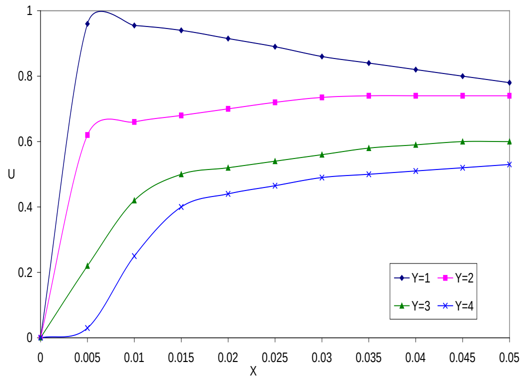

Figure 9: Velocity profile for Y and K = 8

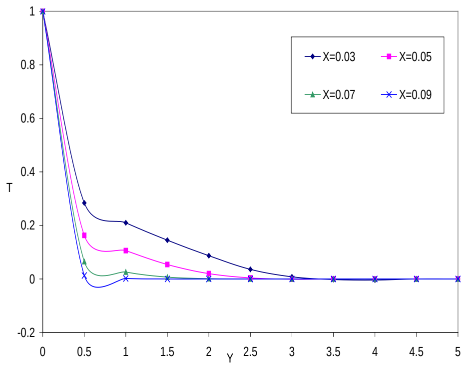

Figure 10: Temperature profile for X and K = 0

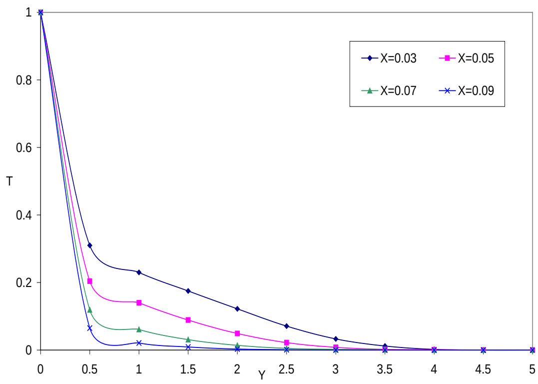

Figure 11: Temperature profile for X and K = 2

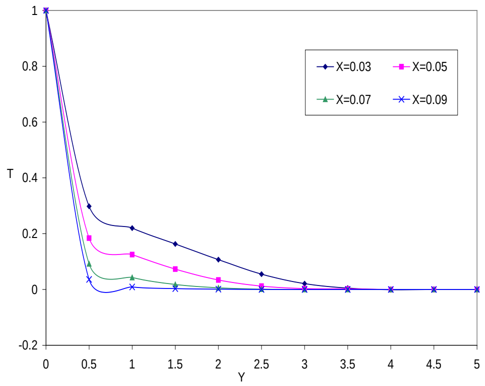

Figure 12: Temperature profile for X and K = 4

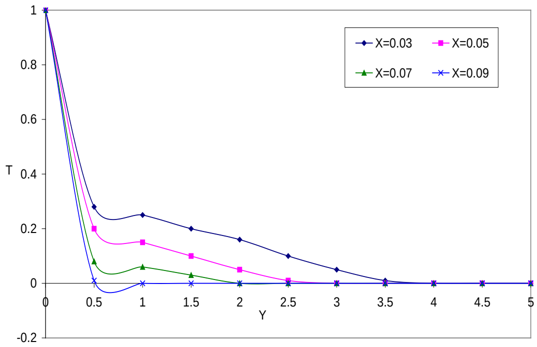

Figure 13: Temperature profile for X and K = 8

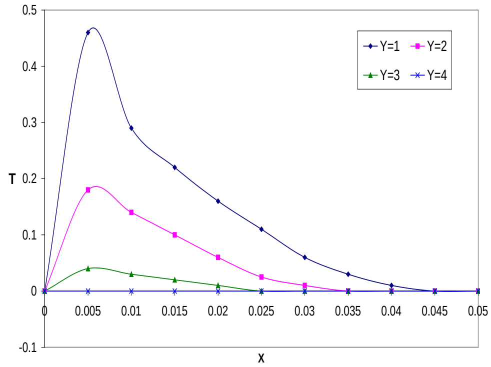

Figure 14: Temperature profile for Y and K = 0

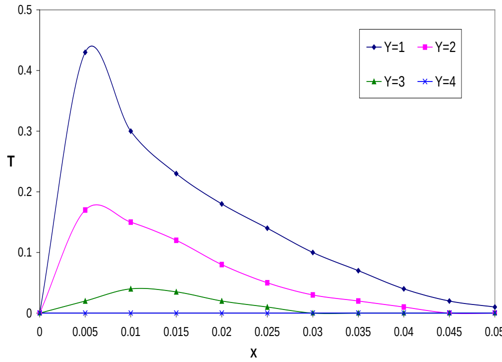

Figure 15: Temperature profile for Y and K = 2

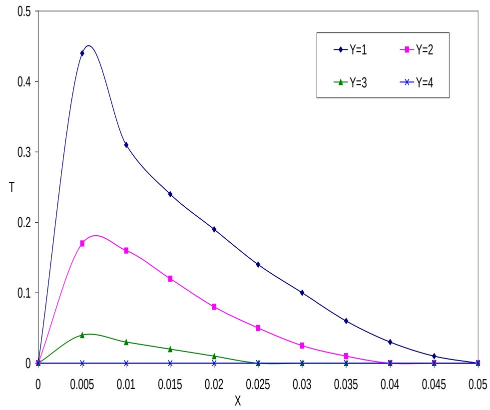

Figure 16: Temperature profile for Y and K = 4

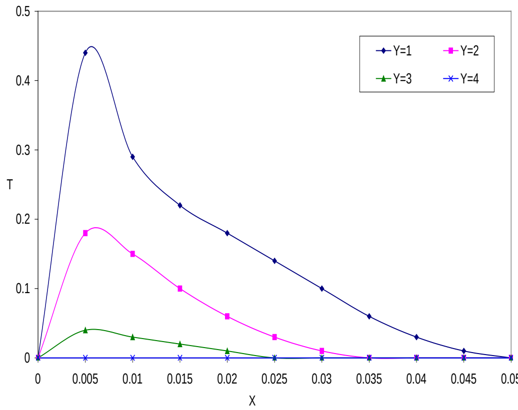

Figure 17: Temperature profile for Y and K = 8

BIBLIOGRAPHY

[1] Bejan, A. and Khair, K.R. Int. J. Heat Mass Transfer, Vol.28, p.909, 1985.

[2] Bejan, A. Int. J. Heat Mass Transfer, Vol.23, p.726, 1980.

[3] Bejan, A. Lett. Heat Mass Transfer, Vol.6, p.93, 1979.

[4] Berezovsky, A.A., Martynenko, O.G., and Yu, A.S., J. Engg. Phys. No.33, pp.32-39, 1977.

[5] Bergholz, R.F. ASME J. Heat Transfer, Vol.102, p.242, 1980.

[6] Blythe, P.A., daniels, P.G., and Simpkins, P.G. Proc. R.Soc. London, A 380, p.119, 1982.

[7] Brinkman, H.C. Appl. Sci. Res. Al, p.27, 1947.

[8] Cebect, T., and Bradshaw, P. Physical and Computational Aspects of convective heat transfer, Springer – Verlag, New York, P. 406, 1984.

[9] Chen, C.K. Hung, C.I., and Cleaver, J.W. Proc. ASME – JSME Thermal Eng. Joint Conf. Vol.2, p.313, 1987.

[10] Cheng, P. Advances in Heat Transfer, Vol.14, P. 1, 1978.

[11] Cheng, P., and Min Knowyz, W.J. J. Geophys, Res., Vol.82, p.2040, 1977.

[12] Gill, A.E. J. Fluid Mech. Vol.26, p515, 1966.

[13] Ingham, D.B. Merkam, J.H. and Pop. I. Int. J. Heat Mass Transfer, Vol.25, P. 1916, 1982.

[14] Ingham, D.B. Proc. R. Soc. London, Ser. A, Vol.402, No.1822, P.109, 1985.

[15] Johnson, C.H., and Cheng, P. Int J. Heat Mass Transfer, Vol.21, P.709, 1978.

[16] Kaviany, M. ASME J. Heat Transfer, Vol.109, p.345, 1987.

[17] Kimura, S., and Bejan, A. ASME J. Heat Transfer, Vol. 107, P.293, 1985.

[18] Kumari , M. Pop.I., and Nath, G. Int. Communs, Heat and Mass Transfer, Vol. 12, P.317, 1985.

[19] Martynenko, O.G., Berezosky, A.A., Yu, A.S., Int. J. Heat Mass Transfer, No.27, pp.869-881, 1984.

[20] Minkowycz, W.J., and Cheng, P. Int. J. Heat and Mass Transfer, Vol. 19, P.805, 1976.

[21] Muskat, M. The flow of homogeneous fluids through porous Media, Edwards, M.L., 1946.

[22] Ostrach, S. NACA TN 3141, 1954.

[23] Ostrach, S. Trans. ASME, No.75, pp.1287-1290, 1953.

[24] Plumb, O.A., and Huenefelf, J.C. Int, J. Heat Mass Transfer, Vol.24, P.765, 1981.

[25] Prasad, V. Kulacki, F.A., and Keyhani, M. Fluid Mech. Vol.150, P.89, 1985.

[26] Punyatma Sing, Abha Rani, And Ajay kumar Sing, Proc. Of 15th Nat. Conf. On Fluid Mechanics and Fluid Power, REC, Srinagar, Kashmir, 1987.

[27] Raptis et.al. A.A., and Vlahos, J. Lett Heat Mass Transfer, Vol.9, P.59, 1982.

[28] Siegel, R., Tran. ASME, No.30, pp.347-359, 1958.

[29] Simpkins, P.G., and Blythe, P.A. Int. J. Heat Mass Transfer, Vol.124, P.497, 1981.

[30] Simpkins, P.G., and Blythe, P.A. Int. J. Heat Mass Transfer, Vol.123, P.88, 1980.

[31] Soundalgakar, V.M. Trans ASME J. Heat Mass Transfer, Vol.99, P.499, 1977.

[32] Soundalgekar, V.M., Wavre, P.D., Int. J. Heat Mass Transfer, No.20, pp.1363-1373, 1977.

[33] Sparrow, E.M., Beavers, G.S. and Grass, J.I. Physics of Fluids, Vol.2, P.319, 1959.

[34] Tong, T.W. and Subramanian, E. Int. J. Heat Mass Transfer, Vol.28, No.3, P.563, 1985.

[35] Vafai, K., and Tien, C.L. J. Fluid Mech., Vol.85, P.527, 1981.

[36] Weber, J.E. Int. J. Heat Mass Transfer, Vol.18, P.569, 1975.

[37] Wooding, R.A. J. Fluid Mech. Vol.15, P.527, 1963.

[38] Yamamoto, K. and Iwamura, N. J. Engg. Math., Vol.10, No.1, P.41, 1976.