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A COSMOLOGICAL MODEL WITH DECELERATION PARAMETER AND VARYING Λ

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Abstract: *Einstein's field equations are obtained for Bianchi type-I space-time filled with perfect fluid with variable G and Λ by assuming the special law of variation for Hubble's parameter. A particular value of the deceleration parameter was obtained . physical consequences of the model are discussed in the case of Zel'dovich fluid*

Keywords: *Bianchi type-I, Hubble's parameter, deceleration parameter.*

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1. INTRODUCTION:

At the present state of evolution, the Universe is spherically symmetric and the matter distribution in it is on the whole isotropic and homogeneous. But in its early stages of evolution, it could not have had such a smoothed out picture, so the forms of matter fields in the early Universe are uncertain. Friedmann-Robertson-Walker (FRW) models, being isotropic and homogeneous, represent best the large-scale structure of the present Universe.But to describe the early stages of the evolution of the Universe, models with anisotropic background are suitable. The simplest anisotropic models of the universe are Bianchi type-I homogeneous models For a simplification and description of the large scale structure and behaviour of the actual Universe, anisotropic Bianchi type I models have been considered by a number of authors.

Solutions to the field equations may also be generated by applying a law of variation for Hubble's parameter which was proposed by Berman (1983), that yields a constant value of deceleration parameter.Cosmological models with a constant deceleration parameter have been studied by Berman (1983),, Berman and Gomide (1988), Beesham (1993), and others. Saha (2005,2006) has investigated in a series of papers such evolution of Bianchi type-I space-time in the presence of perfect fluid as well as viscous fluid. To consider a jointly the variation of G and Λ within the framework of general relativity has been introduced recently by Berman (1991),Bheesham(1986) singh (2006a).Recently, the present author studied Bianchi type-I cosmological models with time dependent G and Λ (2012). The simplicity of the field equations and relative easy of solution made Bianchi space-time useful in constructing models of spatially homogeneous and an isotropic cosmologies.

In this paper, we consider the space-time to be of the Bianchi type-I with variable G and Λ in the presence of a perfect fluid. In order to solve the field equations, we apply a law of variation for Hubble's parameter . This law, together with the Einstein's field equations, leads to a new solution of the Bianchi type-I space-time.The physical behaviour of the models is discussed in detail.

2. METRIC AND FILED EQUATIONS:

We use here the spatially homogeneous and anisotropic Bianchi type-I line element in the form

$$ds^2 = -dt^2 + A^2(t)\, dx^2 + B^2(t)\, dy^2 + C^2(t)\, dz^2 \qquad .....(1)$$

We assume that the cosmic matter is represented by the energy momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)\, v_i v_j + p g_{ij} \qquad .....(2)$$

where ρ is the energy density of the cosmic matter and p is its pressure, v_i is the four velocity vector such that $v_i v^i = 1$.

The Einstein's field equations with time dependent G and Λ given by (Weinberg 1972)

$$R_{ij} - \tfrac{1}{2} R g_{ij} = -8\pi\, G(t)\, T_{ij} + \Lambda(t) g_{ij} \qquad .....(3)$$

For the metric (1) and energy - momentum tensor (2) in co-moving system of co-ordinates, the field equation (3) yields.

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G p + \Lambda \qquad ....(4)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -8\pi G p + \Lambda \qquad ....(5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G p + \Lambda \qquad ....(6)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = 8\pi G \rho + \Lambda \qquad ....(7)$$

In view of vanishing divergence of Einstein tensor, we have

$$8\pi G\left[\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right] + 8\pi\rho\dot{G} + \dot{\Lambda} = 0 \qquad ....(8)$$

The usual energy conservation equation $T^{j}_{i;j} = 0,$ yields

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \qquad ....(9)$$

Equation (8) together with (9) puts G and Λ in some sort of coupled field given by

$$8\pi\rho\dot{G} + \dot{\Lambda} = 0 \qquad ....(10)$$

Here and elsewhere a dot denotes for ordinary differentiation with respect to t.

Let R be the average scale factor of Bianchi type -I universe i.e.

$$R^3 = ABC \qquad .....(11)$$

From (5), (6) and (7), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{R^3} \qquad .....(12)$$

and $$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{R^3} \qquad .....(13)$$

where k_1 and k_2 are constant of integration. The Hubble parameter H, volume expansion θ, deceleration parameter q are given by

$$H = \frac{\theta}{3} = \frac{\dot{R}}{R} \qquad (14)$$

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{-R\ddot{R}}{\dot{R}^2} \qquad (15)$$

Equations (5)-(8) and (10) can be written in terms of H, σ and q as

$$H^2(2q-1) - \sigma^2 = 8\pi Gp - \Lambda \qquad .....(16)$$

$$3H^2 - \sigma^2 = 8\pi G\rho + \Lambda \qquad .....(17)$$

We assume that the matter content obeys the equation of state

$p = \omega\rho$

Here we take $\omega=1$. .....(18)

The non-vanishing components of the shear tensor σ_{ij} defined by

$$\sigma_{ij} = u_{i;j} + u_{j;i} - \frac{2}{3} g_{ij} u;_k{}^k \qquad .....(19)$$

$$\sigma^2 = \frac{1}{3}\left[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA}\right)\right] \qquad .....(20)$$

$$\frac{\dot{\sigma}}{\sigma} = -3H \qquad ....(21)$$

3. SOLUTION OF THE FIELD EQUATIONS:

The system of equations (4)-(7), (11) and (18) supply only six equations in seven unknowns (A,B,C, ρ, p, G and Λ).One extra equation is needed to solve the system completely.Therefore we first take that Hα R^{-1} ,i.e.

$$H = \frac{a}{R} \qquad .....(22)$$

where a is positive constants

From equ.(14)and (22) we get

R=(at+c_1) where c_1 is constant of integration ....(23)

Substituting (23) into (15), we get

$$q=0 \qquad ....(24)$$

using Eqs. (12),(13) and (23), we obtain the line-element (1) in the form

$$ds^2 = -dt^2 + (at+c_1)^2 \exp\left[\frac{-(2k_1+k_2)}{3a}(at+c_1)^{-2}\right]dx^2 + (at+c_1)^2 \exp\left[\frac{k_2-k_1}{3a}(at+c_1)^{-2}\right]dy^2$$

$$+(at+c_1)^2 \exp\left[\frac{(k_1+2k_2)}{3a}(at+c_1)^{-2}\right]dz^2 \qquad (25$$

For the model (25), we have

$$V = (at+c_1) \qquad .....(26)$$

$$\rho=p=\frac{k_3}{(at+c_1)^6} \qquad .....(27)$$

$$\sigma=\sqrt{k_4}\frac{1}{(at+c_1)^3} \qquad .....(28)$$

$$G=\frac{1}{8\pi k_3}\left[\frac{a^2}{(at+c_1)^{-4}} - k_4\right] \qquad .....(29)$$

$$\Lambda=\frac{4a^2}{(at+c_1)^2} \qquad .....(30)$$

$$\theta = \frac{3a}{(at+c_1)} \qquad .....(31)$$

CONCLUSION

We observe that the spatial volume V is zero at t = $-\frac{c_1}{a}$ = t_0 (say), and expansion scalar θ is infinite at t = t_0 which shows that the universe starts evolving with zero volume and infinite rate of expansion at t = t_0. Initially, at t = t_0, the spacetime exhibits a 'point type' singularity. At t = t_0, ρ, p, Λ, σ, G are all infinite. As t increases the spatial volume increases, but the expansion scalar decreases. Thus, the expansion rate decreases as the time increases.

As t $\to \infty$, the spatial volume V becomes infinitely large. All parameters, ρ, p, Λ, σ ,θ tend to zero, and G is constant at late times. Therefore, the model essentially gives an empty universe for large t. The ratio $\frac{\sigma}{\theta} \to 0$ as t$\to \infty$,which shows that the model approaches isotropy for large value of t. We see that Λ is positive and also $\Lambda \alpha \frac{1}{t^2}$, i.e., Λ is a decreasing function of time(Berman,1991) . This supports the results obtained from recent supernova Ia observations (Singh CP et all). Also, G is a decreasing function of time and becomes negligible for large t. Therefore, the model represents shearing, non-rotating and expanding universe with a big-bang.

REFERENCES

1. Beesham, A. Int. J. Theor. Phys. 25 1295(1986) Preprint, University of Zululand (1993)..
2. Berman M. S. and Gomide, F. M. Gen. Rel. Grav. 20 191(1988).
3. Berman, M. S. Gen. Relativ. Gravit. 23 465 (1991)
4. Berman, M. S. Nuovo Cimento B 74 182(1983).
5. Berman, M. S. Gen. Relativ. Gravit. 23 465(1991).
6. Saha, B. Chinese J. Phys. 43 (6) 1035(2005).
7. Saha, B. Int. J. Theo. Phys. 45 (2006) 5.
8. Singh, C. P. Mod. Phys. Lett. A 21 1803 (2006a)
9. Singh, CP, Kumar, S., Pradhan, A.: Class Quantum Gravity 24, 455 (2007).