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BIANCHI TYPE-I COSMOLOGICAL MODEL WITH TIME VARYING G AND Λ IN GENERAL RELATIVITY

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Abstract: *Einstein's field equations with variable cosmological constants are considered in the presence of stiff fluid for Bianchi type-I universe by assuming the cosmological term proportional to R^{-1} (R is scale factor). I obtain that the present universe is accelerating with a large fraction of cosmological density in the from of cosmological term .The physical significance of the cosmological models are also discussed.*

Keywords: *Bianchi type-I,Variable cosmological term, Stiff fluid.*

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1. INTRODUCTION:

Bianchi type-I space-time is the simplest generalization of the Friedmann-Robertson-Walker (FRW) flat models. There is a significant observational evidence that the expansion of the universe in undergoing a late-time acceleration (Padmanabhan 2003 Lima 2004) .It is also well known that there is a certain degree of anisotropy in the actual universe. Therefore, we have chosen the metric for the cosmological model to be Bianchi type-I.

Cosmological scenarios with a time-varying Λ were proposed by several researchers.A number of models with different decay laws for the variation of cosmological term were investigated during the last two decades.Chen and Wu(1990), Pavan (1991), Carvalho et al. (1992), Lima and Maia (1994), Lima and Trodden(1994), , Arbab and Abdel-Rahman(1994), , Cunha and Santos (2004), Carneiro and Lima (2005), A lot of work has been done by Saha (2005,2006)in studying the anisotropic Bianchi type-I cosmological model in general relativity with varying G and Λ.

In this paper I have considered a Bianchi type-I cosmological model with variables G and Λ filled with stiff fluid . I obtain solution of the Einstein field equations by assuming the cosmological term proportional to R^{-1} (where R is scale factor). The paper is organized as follows.Basic equations of the model are given in Sec. 2 and their solution in Sec. 3. We discuss the model and conclude our results in Sec.4.

2. Metric and Filed Equations:

We use here the spatially homogeneous and anisotropic Bianchi type-I line element in the form

$$ds^2 = -dt^2 + A^2(t)\, dx^2 + B^2(t)\, dy^2 + C^2(t)\, dz^2 \qquad .....(1)$$

We assume that the cosmic matter is represented by the energy momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)\, v_i v_j + p g_{ij} \qquad .....(2)$$

where ρ is the energy density of the cosmic matter and p is its pressure, v_i is the four velocity vector such that $v_i v^i = 1$.

The Einstein's field equations with time dependent G and Λ given by (Weinberg 1972)

$$R_{ij} - ½ R g_{ij} = -8\pi\, G(t)\, T_{ij} + \Lambda(t) g_{ij} \qquad .....(3)$$

For the metric (1) and energy - momentum tensor (2) in co-moving system of co-ordinates, the field equation (3) yields.

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=-8\pi Gp+\Lambda \qquad ....(4)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=-8\pi Gp+\Lambda \qquad ....(5)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=-8\pi Gp+\Lambda \qquad ....(6)$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=8\pi G\rho+\Lambda \qquad ....(7)$$

In view of vanishing divergence of Einstein tensor, we have

$$8\pi G\left[\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right]+8\pi\rho\dot{G}+\dot{\Lambda}=0 \qquad ....(8)$$

The usual energy conservation equation $T^{j}_{i;j}=0$, yields

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \qquad ....(9)$$

Equation (8) together with (9) puts G and Λ in some sort of coupled field given by

$$8\pi\rho\dot{G}+\dot{\Lambda}=0 \qquad ....(10)$$

Here and elsewhere a dot denotes for ordinary differentiation with respect to t.

Let R be the average scale factor of Bianchi type -I universe i.e.

$$R^3 = ABC \qquad .....(11)$$

From (5), (6) and (7), we obtain

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_1}{R^3} \qquad .....(12)$$

and

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{k_2}{R^3} \qquad .....(13)$$

where k_1 and k_2 are constant of integration. The Hubble parameter H, volume expansion θ, sheer σ and deceleration parameter q are given by

$$H=\frac{\theta}{3}=\frac{\dot{R}}{R},\ \sigma=\frac{k}{\sqrt{3}R^3},\ k>0\ \text{(constant)},\ q=-1-\frac{\dot{H}}{H^2}=-\frac{-R\ddot{R}}{\dot{R}^2}$$

Equations (5)-(8) and (10) can be written in terms of H, σ and q as

$$H^2(2q-1)-\sigma^2=8\pi Gp-\Lambda \qquad .....(14)$$

$$3H^2-\sigma^2=8\pi G\rho+\Lambda \qquad .....(15)$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{R}}{R} = 0 \qquad .....(16)$$

From (16), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi G\rho}{\theta^2} - \frac{\Lambda}{\theta^2}$$

Therefore, $0 \le \frac{\sigma^2}{\theta^2} \le \frac{1}{3}$ and $0 \le \frac{8\pi G\rho}{\theta^2} \le \frac{1}{3}$ for $\Lambda \ge 0$

Thus, the presence of positive Λ puts restriction on the upper limit of anisotropy, where as a negative Λ contributes to the anisotropy.

From (16), and (17), we have

$$\frac{d\theta}{dt} = -12\pi Gp - \frac{\theta^2}{2} + \frac{3\Lambda}{2} = 12\pi G(\rho + p) - 3\sigma^2$$

Thus the universe will be in decelerating phase for negative Λ, and for positive Λ, universe will slows down the rate of decrease. Also $\dot{\sigma} = -\frac{3\sigma\dot{R}}{R}$ implying that σ decreases in an evolving universe and it is negligible for infinitely large value of R.

3. SOLUTION OF THE FIELD EQUATIONS:

The system of equations (5)-(8), and (11), supply only five equations in seven unknowns (A,B,C, ρ, p, G and Λ). Two extra equation is needed to solve the system completely.Therefore we first take that $\Lambda \alpha R^{-1}$,i.e.

$$\Lambda = \frac{a}{R} \qquad .....(17)$$

where a is positive constants

This law was proposed by Holy, F. et al (1997) considered $\Lambda\ \alpha\ a^{-3}$ whereas $\Lambda \alpha\ a^{-m}$ (a is scale factor and m is constant) considered by Olson, T.S. et al. (1987), Pavon, D. (1991), Maia, M.D. et al. (1994), Silveria, V. et. al. (1994, 1997) , Torres, LF. B. et al. (1996).

As the second condition, we assume that

$$A=BC \qquad ....(18)$$

This law was proposed by Collins et.al.

Using the condition (18) in (12) and (13), after suitable transformation, the metric (1) takes the following form

$$ds^2 = -dT^2 + Tdx^2 + T^{\left(\frac{K_1+K_2}{2K_1+K_2}\right)}dy^2 + T^{\frac{K_1}{2K_1+K_2}}dz^2$$ (19)

For the model (19)

$$R = T^{\frac{1}{3}}$$ ......(20)

$$\theta = \frac{1}{T},$$ ...(21)

$$\sigma = \frac{k}{\sqrt{3}}\frac{}{T}$$ (22)

$$p = \frac{1}{8\pi k_3}\left[aT^{-\frac{1}{3}} + \frac{k_4}{T^2}\right]\left[k_4 - aT^{\frac{5}{3}}\right]^{\frac{1}{5}}$$ (23)

$$\rho = \frac{1}{8\pi k_3}\left[-aT^{-\frac{1}{3}} + \frac{k_4}{T^2}\right]\left[k_4 - aT^{\frac{5}{3}}\right]^{\frac{1}{5}}$$ (24)

$$G = k_3\left[k_4 - aT^{\frac{5}{3}}\right]^{-\frac{1}{5}}$$ (25)

$$\Lambda = aT^{-\frac{1}{3}}$$ (26)

Where $k_{3,} k_4$ are constant of integration.

CONCLUSION

In the model at T = 0, the energy density ρ, pressure p, shear σ and the cosmological term Λ all are infinite, but G is finite. As T increases, the expansion rate decreases , Λ decreases but G increases . The possibility of G increasing with time has been investigated by Abdel-Rahman (1990),Chow (1981).Clearly, $\frac{\sigma}{\theta} = \frac{k}{\sqrt{3}}$. Therefore, the model does not approach isotropy. we observe that at T = 0 the expansion scalar θ is infinite, which shows that universe starts evolving with zero volume and an infinite rate of expansion at T = 0. The scale factors also vanish at T = 0 and hence the model has a point-type singularity at initial epoch.

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