EMPIRICAL ANALYSIS BASED ON COMPARISON AMONG VARIOUS METHODS OF OBTAINING IBFS TO UNBALANCED TRANSPORTATION PROBLEM

S. S. Kulkarni*

Abstract: *This paper presents a comparative study of various methods of obtaining Initial Basic Feasible Solution (IBFS) to Unbalanced Transportation Problem viz., NWCR, Matrix-minima method (least cost method), VAM, VAMT-TOC, and method given by Kirca-Satir(1990)) with our Cost-Sum method using statistical experimental design. The measure of the quality and the effectiveness for comparison of these methods are average relative deviation and the number of iterations to reach the optimality. It is also presented that how many instances gives the IBFS≡ OBFS cost which proves the superiority of Cost-Sum method as compared to other methods.*

Keywords: *Unbalanced Transportation Problem (UTP), Vogel's approximation method, total opportunity cost, Cost-Sum method, total opportunity cost matrix, IBFS.*

*Head, Department of Statistics, Willlingdon College, Sangli (India)

1. INTRODUCTION

In classical transportation problem transportation schedule is designed so that whatever be the units available are to be transported to the places in such a way that total requirement is satisfied ,such a transportation problem is called balanced transportation problem otherwise it is called unbalanced one. In literature, various methods are available to solve unbalanced transportation problem viz. Cost-Sum method[1] , Zero –Suffix method[2], (Zero point method)[3] ,IVAM[4] ,method suggested by Shimshak et.al.[5], method suggested by Goyal[6], method suggested by Kore [7], method suggested by Kirca-Satir [8], methods suggested by Mathrajan-Meenakshi [9], method suggested by Ramakrishnan[12], etc. In 2004, Mathrajan-Meenakshi have compared the different variants of VAM viz VAM-TOC, VAMT-TOC and VAM for solving large sized problems. To compare the effectiveness of methods they have used different measures of effectiveness such as Average Relative Percentage Deviation [APRD], number of best solutions.

While solving unbalanced transportation problem, classical methods suggests to add a dummy row or dummy column having cost zero to balance the given problem and then apply VAM to solve the transportation problem. Shimshak et.al.[5] have suggested that balance the problem with the conventional method but the penalty costs are ignored not only for dummy column but also for all the rows since the calculation of dummy rows involve the dummy column. Goyal[6] suggested another modification by taking into consideration the costs allocated to the dummy row or dummy column. Dummy row or dummy column is allocated highest cost in the transportation table. Improved Zero-point method given by Samuel[10] is used to solve both crisp and fuzzy transportation problem. Zero –Suffix method takes into consideration the computation of suffix values. Zero Point method is similar to this method.

Kore [7] suggests that there is no need to balance the given transportation problem. Kirca-Satir [8] suggested a heuristic method based on computation of TOM (Total Opportunity Cost Matrix).The TOM is, in fact, an application of the least-cost method along with some different tie-breaking features on the Total Opportunity Cost Matrix (TOC). The TOC Matrix is obtained by adding the ROC (Row Opportunity Cost) matrix to the COC (Column Opportunity Cost) matrix.

Cost- sum method is also a heuristic method that computes the sums of each row and each column and highest sum is taken into consideration for the allocation of units in the corresponding row and column.

In order to obtain optional solution to a transportation problem first it is needed to have Initial Basic Feasible Solution (IBFS). Different methods discussed above are available. In this paper our aim into compare the available methods with the aid of C++ program developed by the author. We present the statistical experiment design which facilities the comparison between various methods, enumerated above. The experiment and analysis of the data are also presented. The main aim of this experiment is to evaluate the quality of various methods of obtaining IBFS to transportation problems. The quality and the effectiveness of the methods were measured in terms of number of iterations required to reach the optional solution and Average Relative Deviation (ARD) and Relative Deviation (RD) derived from the experimental analysis presented below in the section 2.

The paper is organised as follows. In section 2, experimental design of the experiment carried out is given. In section 3, comparison between various methods with the 135 randomly generated examples is carried out.

2. EXPERIMENTAL DESIGN

The performances of different methods are compared over 135 examples. These examples are randomly generated using the following framework.

Problem Size (m × n): The different sizes of the transportation problems that are generated randomly are: (3× 3), (5× 5), and (7× 7)

Cost structure: The problems with three values for the cost-range R are tested. The mean cost is taken to be 50. The ranges used are R = 10, 20, 30. For each range, the costs are randomly generated from the uniform distribution:

$$U(C_{ij}: [\text{mean cost} - R/2, \text{mean cost} + R/2])$$

Supply and demand structure(a_i, b_j): The mean demand taken = 75. Given the mean demand, mean supply is given by: Mean supply= k{(n × mean demand) / m},

where k indicates the degree of imbalance between the total supply and the total demand. The mean supply values are generated for three values of the imbalance coefficient, k, viz. k = 1, 2, 3. The a_i and b_j are then generated from the uniform distributions:

$$U(a_i : 0.75 \times \text{mean supply}, 1.25 \times \text{mean supply})$$

$U(b_j$: 0.75 × mean supply, 1.25 × mean supply)

The experimental design for generating the test problems using the above three parameters is summarized in the Table 2. A C++ program is written for this experimental design. For each combination of the values for [(m × n), k, R] five problem - instances are randomly generated, yielding a total of 135= 3x3x3x5 problems. All these 135 problems are unbalanced transportation problems.

Table 1: Summary of Experimental Design

No.	Problem Factor	Levels	# levels
1	Problem size	(3×3),(5×5), (7×7)	3
2	Degree of imbalance	(1,2,3)	3
3	Cost structure Range (R)	(10,20,30)	3
4	No. of problem configurations		3×3×3=27
5	Problem - instances per configuration		5
6	Total no. of problems		**27×5**=135
Cost structure (C_{ij}) : U(C_{ij} : [mean cost – R/2, mean cost + R/2] .where mean cost=50			
Supply (a_i) : U(a_i : 0.75 × mean supply, 1.25 × mean supply), Demand (b_j) : U(b_j : 0.75 × mean supply, 1.25 × mean supply) where mean demand = 75 and mean supply = [(k × n × mean demand) / m].			

Measure of effectiveness: The performances of the methods may vary over a range of problems under consideration. The performances of different methods are compared using the following two measures

(1) Average Relative Deviation(ARD): The ARD, which indicates the average performance of various methods with respect to the optimal solution is compared over the number of problem-instances. The formula used for this type of comparison is:

$$\mathrm{ARD(H)} = \frac{1}{\mathrm{N}}\sum_{i=1}^{N} RD(H,i)$$

where $\mathrm{RD(H)} = \frac{(\text{IBFS cost} - \text{OBFS cost})}{\text{OBFS cost}}$

Here (i) ARD(H) stands for average relative deviation of the given heuristic method H,

(ii) N stands for the number of problem - instances over which the average is compared

(2) Number of iterations required to reach the optimal solution.

Now we present the conclusions based on the 135 randomly generated examples of UTP.

Table 2 records the number of problem - instances in which IBFS≡OBFS

Table 2: the number of problem - instances in which IBFS≡OBFS

Method	Number of problem- instances in which IBFS≡OBFS
Cost-Sum	1
VAM	13
Total problem- instances	14

It can be seen from this table that the number of problem – instances in which IBFS coincides with the OBFS is the highest for the VAM. Also, no other method needs zero iterations to reach the optimal except our Cost-Sum method. This finding can also be noted from the graph presented in Fig 2.1 on page number 6.

From this it is clear that VAM works much better for UTP.

Remark: Many researchers constantly try to discover different methods to obtain an IBFS to a transportation problem. This has been a continuous activity for a long period of time, and various procedure have come up as a result of these efforts. In this connection it may be worthwhile to quote Hadley, an authority on this subject. He states "Many other techniques for determining an initial solution might have been discussed ..... It is by no means established that any one of the methods is better than the others, ... To decide which method for determining an initial basic feasible solution leads to the smallest number of iterations .....it would be necessary to solve the problem in each case." In view of this remark by Hadley, one may not hope to discover the best or the most superior algorithm to obtain an IBFS to a transportation problem. But one can obtain a satisfaction if one is able to discover a method which is comparatively more successful than many other existing methods from the point of view of certain desirable criteria, and that is what we have achieved in this paper. From the tables and graphs based on the comparison carried out manually as well as on the basis of sufficient number of randomly generated problems, it is clearly seen that our Cost-Sum method to obtain an IBFS to unbalanced transportation problems of six different types has yielded better result in more number of numerical problems and as such as we humbly feel that our contribution in this respect is worthwhile and significant.

Table 3 records the number of iterations (one of the measures of effectiveness) required by six methods used for comparison

Number of problem – instances and its corresponding number of iterations required to reach optimal solution starting with the corresponding IBFS.

Table 3

No. of Iterations	North-West	Matrix-Minima	VAM	Cost-Sum	VAM-TTOC	Kirca-Satir
0	0	0	13	1	0	0
1	4	10	29	7	11	14
2	15	19	28	15	17	17
3	17	22	26	30	24	22
4	11	23	15	26	25	27
5	6	12	11	27	18	14
6	14	19	7	12	13	13
7	11	14	4	14	14	7
8	13	7	2	2	6	15
9	10	4	1	1	6	5
10	8	4	0	0	1	1
11	8	0	1	0	0	0
12	5	0	0	0	0	0
13	3	0	0	0	0	0
14	7	0	0	0	0	0
15	3	0	0	0	0	0
Total	135	135	135	135	135	135

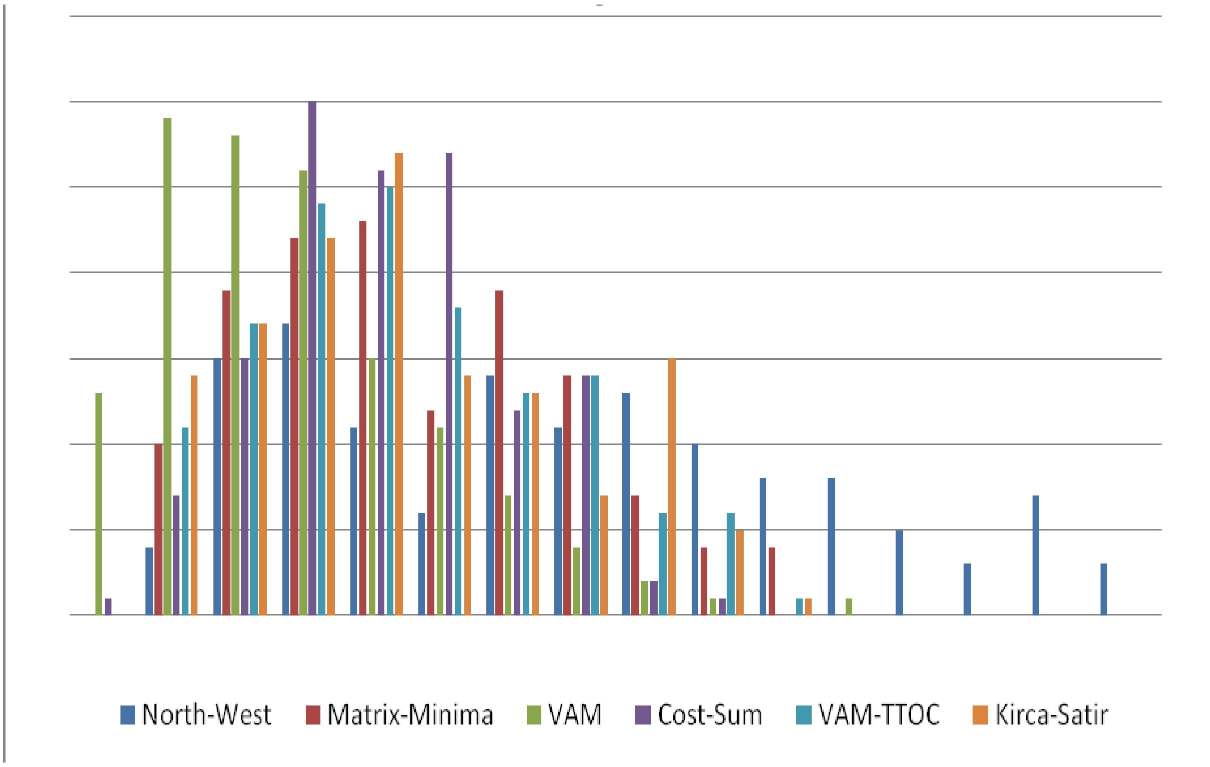

Fig 2.1 Distribution of Iterations for the solved examples of UTP

REFERENCES

1. Kulkarni S. S. (2012): On Initial Basic Feasible Solution for solving Transportation Problem – A New Approach. J. Indian. Acad. Math. Vol.34, No 1, 19-25.
2. Sudhakar V.J., Arunsankar N. & Karpagam T. (2012); A New Approach for finding an Optimal Solution for Transportation Problems, Euro. Jour. of Scientific Research, Vol. 68, No. 2,254-257.
3. Singh Shweta, Dubey G.C., Shrivastava Rajesh (2012); A Various Method of Solve the Optimality for the Transportation Problem, Int. Jour. Mathematical Engineering and Science, Vol. 1 (4), 21-28.
4. Korukogla S and Balli S (2011); An Inproved Vogel's Approximation method for the Transportation Problem, Mathematical and Computational Applications, Vol. 16, No.2, 370-381.
5. Shimshak, DG, JA Kaslik and TD Barclay (1981): A modification of Vogel's approximation method through the use of heuristics. INFOR, 19, 259-263.
6. Goyal, S.K. (1984): Improving VAM for unbalanced transportation problems. Journal of Operational Research Society, 35(12), 1113-1114.
7. Kore B. G. and Thakur L. B. (2000): A New Method for obtaining Optimal Solution of Unbalanced Transportation Problem. J. Indian. Acad. Math. Vol. 22, 247-258.
8. Kirca, O and A Satir (1990): A heuristic for obtaining an initial solution for the transportation problem. Journal of Operations Research Society, 41(9), 865-871.
9. Mathirajan M. And Meenakshi B. (2004): Experimental Analysis of Some Variants of Vogel's Approximation Method. Asia-Pacific J. of O. R., Vol. 21, No. 4, 447-462.
10. Samuel Edward A (2012) ; Improved Zero Point Method (IZPM) for the Transportation Problems, Applied Mathematical Science, Vol. 16, No. 109, 5421-5426.
11. Tikekar V. G., Kulkarni S. S. (2010): On IBFS to Transportation Problems. Bull. Marth. Math. Soc, Vol. 11, No. 1, 72-74.
12. Ramakrishnan, C.S. (1988). An improvement to Goyal's modified VAM for the unbalanced transportation problem. Journal of Operational Research Society, 39(6), 609 -610.
13. Hadley, G. (1962): Linear Programming. Addison-Wesley Pub. pp. 437-447.
14. Imam T., Eisharawy G., Gomah M. (2009); solving Transportation Problem Using Object-Oriented Model, Inf. Jour. of Computer Science and network Security Vol. 9, no. 2, 353-359.
15. Deshmukh N.M. (2012); An Innovative Method for Solving Transportation Problem, Int. Jour, of Physics and Mathematical Science, Vol. 2(3), 86-91.