SELECTION OF MIXED SAMPLING PLAN WITH QSS-2($n;c_N,c_T$) PLAN AS ATTRIBUTE PLAN INDEXED THROUGH MAPD AND LQL

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Abstract: *This paper presents the procedure for the construction and selection of the mixed sampling plan using MAPD as a quality standard with the QSS-2 ($n;c_N,c_T$) plan as attribute plan. The plans indexed through MAPD and LQL are constructed and compared for their efficiency. Tables are constructed for easy selection of the plan.*

Keywords and Phrases: *Limiting quality level, maximum allowable percent defective, operating characteristic, tangent intercept.*

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1. INTRODUCTION

Mixed sampling plans consist of two stages of rather different nature. During the first stage the given lot is considered as a sample from the respective production process and a criterion by variables is used to check process quality. If process quality is judged to be sufficiently good, the lot is accepted. Otherwise the second stage of the sampling plan is entered and lot quality is checked directly by means of an attribute sampling plan.

There are two types of mixed sampling plans called independent and dependent plans. If the first stage sample results are not utilized in the second stage, the plan is said to be independent otherwise dependent. The principal advantage of a mixed sampling plan over pure attribute sampling plans is a reduction in sample size for a similar amount of protection.

The second stage attribute inspection becomes more important to discriminate the lot if the first stage variable inspection fails to accept the lot. If rejection occurs during the normal inspection, tightened inspection is recommended in the mixed system and vice versa in the second stage. Hence Quick Switching System is imposed in the second stage to sharpen the sampling situation and to insist the producer to manufacture goods within the Limiting Quality Level. Dodge (1967) proposed a sampling system called a 'Quick Switching System' (QSS) consisting of pairs of normal and tightened plans.

Schilling (1967) proposed a method for determining the operating characteristics of mixed variables –attributes sampling plans, single sided specification and standard deviation known using the normal approximation. Devaarul (2003), Sampath Kumar (2007), Sampath Kumar, Indra and Radhakrishnan (2012a, 2012b, 2012c, 2012d) have made contributions to mixed sampling plans for independent case. Sampath Kumar et.al (2012) studied mixed sampling plan for independent case. QSSs were originally proposed by Dodge (1967) and investigated by Romboski (1969) and Govindaraju (1991).Dodge (1967) proposed a new sampling system consisting of pairs of normal and tightened plans. QSS developed with attributes by Romboski (1969) is a reduction in the sample size required to achieve approximately the same operating characteristic curve.

In this paper, using the operating procedure of mixed sampling plan with QSS-2($n;c_N,c_T$) plan as attribute plan, tables are constructed for the mixed sampling plan indexed through (i)

MAPD (ii) LQL (limiting quality level). This plan indexed through MAPD is compared with the plan indexed through LQL. Suitable suggestions are also provided for future research.

2. GLOSSARY OF SYMBOLS

The symbols used in this paper are as follows:

p : submitted quality of lot or process

$P_a(p)$: probability of acceptance for given quality 'p'

p_2 : the submitted quality such that $P_a(p_2) = 0.10$ (also called LQL)

p_* : maximum allowable percent defective (MAPD)

h_* : relative slope at 'p_*'

n_1 : sample size of variable sampling plan

n_2 : sample size of attribute sampling plan

c_N : acceptance number of normal inspection

c_T : acceptance number of tightened inspection

β_j : probability of acceptance for lot quality 'p_j'

β_j' : probability of acceptance assigned to first stage for percent defective 'p_j'

β_j'' : probability of acceptance assigned to second stage for percent defective 'p_j'

d : observed number of nonconforming units in a sample of n units

$z(j)$: 'z' value for the j^{th} ordered observation

k : variable factor such that a lot is accepted if $\bar{X} \leq A = U - k\sigma$

3. OPERATING PROCEDURE OF MIXED SAMPLING PLAN QSS-2(N;C_N,C_T) AS ATTRIBUTE PLAN

Schilling (1967) has given the following procedure for the independent mixed sampling plan with Upper specification limit (U) and known standard deviation (σ).

- Determine the parameters of the mixed sampling plan n_1, n_2, k, c_N and c_T.
- Take a random sample of size n_1 from the lot.
- If a sample average $\bar{X} \leq A = U - k\sigma$, accept the lot.
- If a sample average $\bar{X} > A = U - k\sigma$, go to step 1.

Step 1: From a lot, take a random sample of size n_2 at the normal level. Count the number of defectives 'd'

- If $d \leq c_N$, accept the lot and repeat step 1.

- If $d > c_N$, reject the lot and go to step 2.

Step 2: From the next lot, take a random sample of size n_2 at the tightened level. Count the number of defectives 'd'

- If $d \leq c_T$, accept the lot and use step 1 for the next lot.
- If $d > c_T$, reject the lot and repeat step 2 for the next lot.

4. CONSTRUCTION OF MIXED SAMPLING PLAN HAVING QSS-2(N;C_N,C_T) AS ATTRIBUTE PLAN

The operation of mixed sampling plans can be properly assessed by the OC curve for given values of the fraction defective. The development of mixed sampling plans and the subsequent discussions are limited only to the upper specification limit 'U'. A parallel discussion can be made for lower specification limits.

The procedure for the construction of mixed sampling plans is provided by Schilling (1967) for a given 'n_1' and a point 'p_j' on the OC curve is given below.

- Assume that the mixed sampling plan is independent
- Split the probability of acceptance (β_j) determining the probability of acceptance that will be assigned to the first stage. Let it be β_j'.
- Decide the sample size n_1 (for variable sampling plan) to be used
- Calculate the acceptance limit for the variable sampling plan as $A = U - k\sigma = U - [z(p_j) + \{z(\beta_j')/\sqrt{n_1}\}]\sigma$, where U is the upper specification limit and z (t) is the standard normal variate corresponding to't' such that $t = \int_{z(t)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$
- Determine the sample average $\bar{X}$. If a sample average $\bar{X} > A = U - k\sigma$, take a second stage sample of size 'n_2' using attribute sampling plan.
- Split the probability of acceptance β_j as β_j' and β_j'', such that $\beta_j = \beta_j' + (1-\beta_j')\beta_j''$. Fix the value of β_j'.
- Now determine β_j'', the probability of acceptance assigned to the attributes plan associated with the second stage sample as $\beta_j'' = (\beta_j - \beta_j') / (1-\beta_j')$
- Determine the appropriate second stage sample of size 'n_2' from

$$Pa(p) = \beta_j'' \text{ for } p = p_j.$$

Using the above procedure tables can be constructed to facilitate easy selection of mixed sampling plan with QSS-2(n;c_N,c_T) plan as attribute plan indexed through MAPD and LQL.

According to Soundararajan and Arumainayagam (1988), the operating characteristic functions of QSS-2 is given below.

$$P_a(p) = \frac{ab^2 + b(1-a)(1+b)}{b^2 + (1-a)(1+b)} \quad (1)$$

Where
$$a = \sum_{i=0}^{c_N} \frac{e^{-n_2 p}(n_2 p)^i}{i!} \quad (2)$$

$$b = \sum_{j=0}^{c_T} \frac{e^{-n_2 p}(n_2 p)^j}{j!} \quad (3)$$

(for acceptance number tightening)

5. CONSTRUCTION OF THE PLANS INDEXED THROUGH MAPD

MAPD (p_*), introduced by Mayer (1967) and further studied by Soundararajan (1975) is the quality level corresponding to the inflection point of the OC curve. The degree of sharpness of inspection about this quality level 'p_*' is measured by 'p_t', the point at which the tangent to the OC curve at the inflection point cuts the proportion defective axis. For designing, Soundararajan (1975) proposed a selection procedure for mixed sampling plan indexed with MAPD and R = $\frac{p_t}{p_*}$.

Using the probability mass function of QSS-2, given in expression (1), the inflection point (p_*) is obtained by using $\frac{d^2 p_a(p)}{dp^2}$ = 0 and $\frac{d^3 p_a(p)}{dp^3} \neq 0$ .The relative slope of the OC curve $h_* = \left[\frac{-p}{p_a(p)}\right]\frac{dp_a(p)}{dp}$ at p = p_*. The inflection tangent of the OC curve cuts the 'p' axis at $p_t = p_* + (p_*/h_*)$. The values of n_2p_*, h_*, n_2p_t and $R = \frac{p_t}{p_*}$ are calculated for different values of 'c_N' and 'c_T' for $\beta_*' = 0.04$ using c++ program and presented in Table 1.

Selection of the Plan

For the given values of p_* and p_t, one can find the ratio $R = \frac{p_t}{p_*}$.Find the value in Table 1 under the column R which is equal to (or) just greater than the specified ratio is found and the corresponding value of c_N, c_T and np_* values are noted and the value of n_2 is obtained using $n_2 = \frac{n_2 p_*}{p_*}$.

Example 1: Given $p_* = 0.019$, $p_t = 0.022$ and $\beta_*' = 0.04$, the ratio $R = \frac{p_t}{p_*} = 1.1579$.In Table 1, the nearest R value is 1.1582 which is corresponding to $c_N = 9$ and $c_T = 0$. The value of $n_2p_* = 3.1773$ is found and hence the value of n_2 is determined as $n_2 = \frac{n_2 p_*}{p_*} = \frac{3.1773}{0.019} = 167$. Thus $n_2 = 167$, $c_N = 9$ and $c_T = 0$ are the parameters selected for the mixed sampling plan having QSS-2($n;c_N,c_T$) as attribute plan using Poisson distribution as a baseline distribution, for the given values of $p_* = 0.019$and $p_t = 0.022$.

6. CONSTRUCTION OF MIXED SAMPLING PLAN INDEXED THROUGH LQL

The procedure given in Section 5 is used for constructing the mixed sampling plan indexed through LQL (p_2). By assuming the probability of acceptance of the lot be $\beta_2 = 0.10$ and $\beta_2' = 0.04$, the n_2p_2 values are calculated for different values of 'c_N' and 'c_T' using c++ program and is presented in Table 1.

Selection of the Plan

Table 1 is used to construct the plans when LQL (p_2), 'c_N' and 'c_T' are given. For any given values of p_0, 'c_N' and 'c_T' one can determine n_2 value using $n_2 = \frac{n_2 p_2}{p_2}$.

Example 2: Given $p_0 = 0.05$, $c_N = 7$ and $c_T = 2$ and $\beta_2' = 0.04$.Using Table 1, find $n_2 = \frac{n_2 p_2}{p_2} = \frac{6.1514}{0.05} = 123$. Thus $n_2 = 123$, $c_N = 7$ and $c_T = 2$ are the parameters selected for the mixed sampling plan having QSS-2($n;c_N,c_T$) as attribute plan for a specified $p_0 = 0.05$,$c_N = 7$ and $c_T = 2$.

Table 1: Various characteristics of the mixed sampling plan when $\beta_*' = \beta_2' = 0.04$ and $\beta_2 = 0.10$

c_N	c_T	n_2p_2	β_*''	n_2p_*	h_*	n_2p_t	$R = p_t/p_*$
1	0	2.7853	0.6924	0.6774	0.7448	1.5869	2.3426
2	1	4.4819	0.6547	1.5719	1.0477	3.0722	1.9545
3	2	5.9964	0.6339	2.5131	1.2882	4.4640	1.7763
2	0	2.8143	0.6351	1.0458	1.3445	1.8236	1.7437
3	1	4.5016	0.6223	1.9740	1.5854	3.2191	1.6307
4	4	8.7891	0.5365	4.4768	1.5878	7.2963	1.6298
4	2	6.0125	0.6131	2.9170	1.7879	4.5485	1.5593

5	3	7.4374	0.6061	3.8714	1.9667	5.8399	1.5085
3	0	2.8694	0.6009	1.3754	1.9876	2.0674	1.5031
5	6	11.4285	0.4420	6.8695	2.0465	10.2262	1.4886
4	1	4.5370	0.5983	2.3406	2.1774	3.4156	1.4593
5	2	6.0399	0.5952	3.2969	2.3447	4.7030	1.4265
9	9	15.2376	0.4358	10.1785	2.8663	13.7296	1.3489
6	5	10.1318	0.3697	6.6602	2.9272	8.9355	1.3416
6	3	7.4605	0.4580	4.6569	3.1920	6.1158	1.3133
6	4	8.8090	0.3171	6.0681	3.4269	7.8388	1.2918
9	8	13.9915	0.3501	9.9948	3.6052	12.7671	1.2774
5	1	4.5956	0.3772	3.0494	3.9255	3.8262	1.2547
7	3	7.4966	0.4210	5.0612	4.0276	6.3178	1.2483
10	4	8.9719	0.5575	6.3339	4.4666	7.7520	1.2239
8	3	7.5509	0.4140	5.3753	4.7914	6.4972	1.2087
7	2	6.1514	0.3893	4.3857	4.8151	5.2965	1.2077
7	0	3.4843	0.5342	2.5915	4.8501	3.1258	1.2062
11	4	9.0623	0.5483	6.6935	5.1521	7.9927	1.1941
8	0	3.7148	0.5257	2.8856	5.5902	3.4018	1.1789
9	1	5.1736	0.5305	3.9979	5.6890	4.7006	1.1758
9	0	3.9605	0.5193	3.1773	6.3218	3.6799	1.1582
10	0	4.2167	0.5140	3.4678	7.0521	3.9595	1.1418
12	0	4.7485	0.5066	4.0449	8.4850	4.5216	1.1179
14	1	6.4240	0.5052	5.5483	9.3895	6.1392	1.1065

7. COMPARISON OF MIXED SAMPLING PLAN INDEXED THROUGH MAPD AND LQL

In this section mixed sampling plan indexed through MAPD is compared with mixed sampling plan indexed through LQL by fixing the parameters c_N, c_T and β_j'.

For the specified values of p_* and p_t with the assumption $\beta_*' = 0.04$ one can find the values of c_N, c_T and n_2 indexed through MAPD. By fixing the values of c_N and c_T, find the value of p_0 by equating Pa (p) = $\beta_2 = 0.10$. Using $\beta_2' = 0.04$, c_N and c_T one can find the value of n_2 using $n_2 = \frac{n_2 p_2}{p_2}$ from Table 1. For different combinations of p_*, p_t, c_N and c_T the values of n_2 (indexed through MAPD) and n_2 (indexed through LQL) are calculated and presented in Table 2.

Construction of OC curve

Table 2: Comparison of plans indexed through MAPD and LQL

p_*	p_t	c_N	c_T	INDEXED THROUGH MAPD	INDEXED THROGH LQL
				n_2	n_2
0.017*	0.020	8	0	170	179
0.022	0.032	4	1	106	119
0.026	0.039	3	0	53	61
0.040	0.078	2	1	39	45

*** OC curves are drawn**

The OC curves for the plans $n_2 = 170$, $c_N = 8$, $c_T = 0$ (indexed through MAPD) and $n_2 = 179$, $c_N = 8$, $c_T = 0$ (indexed through LQL) based on the different values n_2p_2 and $p_a(p)$ are presented in Figure 1.

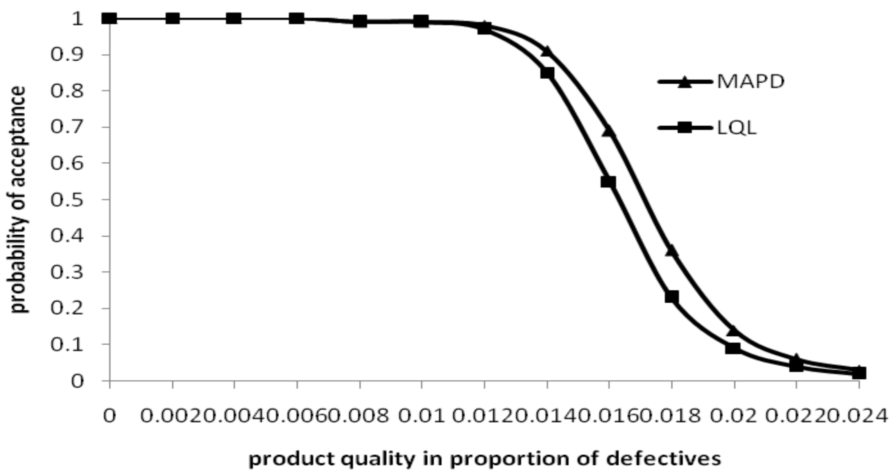

Figure 1: OC Curves for QSS-2(170;8,0) and (179;8,0)

8. CONCLUSION

In this paper, using the operating procedure of mixed sampling plan with QSS-2($n;c_N,c_T$) as attribute plan, tables are constructed for the mixed sampling plan indexed through the parameters MAPD and LQL by taking Poisson distribution as a baseline distribution. It is concluded from the study that the second stage sample size required for QSS-2($n;c_{N,}c_T$) plan indexed through MAPD is less than that of the second stage sample size of the QSS-2($n;c_{N,}c_T$) plan indexed through LQL. Examples are provided for a specified value of $\beta_j' = 0.04$. If the floor engineers know the levels of MAPD or LQL, they can have their sampling plans on the floor itself by referring to the tables. This provides the flexibility to the floor engineers in deciding their sampling plans. Various plans can also be constructed to make the system user friendly by changing the first stage probabilities (β_*', β_2') and can also be compared for their efficiency.

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