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LAMINAR MHD FLOW IN THE ENTRANCE REGION OF A PLANE CHANNEL

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Abstract: *In this paper we investigate laminar MHD flow in the entrance region of plane channel, under the influence of transverse magnetic field. The motion of laminar conducting fluid between two parallel plates is considered. The channel has height of 2a and a width of d. A rectangular coordinate system is placed on channel with the origin at the center of channel at the left hand edge. The flow problem is described by means of partial differential equations and the solutions are obtained by the use of an implicit finite difference technique. The velocity, temperature and pressure profiles are shown in graphs and their behavior is discussed for the different values of magnetic field parameter M and fixed Prandtl Number* P_r.

Keywords:* L*aminar Flow, MHD, Plane Channel.

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1. INTRODUCTION

The effect of magnetic field on the laminar flow of an incompressible electrically conducting fluid is an important problem and has been studied by many investigators. Applications are found in the casting of metals and the growth of semiconductor crystals or cooling fusion devices. It has also many practical applications in ducts and between parallel plates as MHD generators, pumps, accelerators and flow meters. Heat and momentum transfer from a heated moving surface to a quiescent ambient medium occur in manufacturing processes such as hot rolling, wire drawing and crystal growing. The heat treatment of materials traveling between a feed roll and a wind-up roll or on conveyor belts, the lamination and melt-spinning processes in the extraction of polymers possess the characteristics of moving continuous surfaces. MHD finds applications in ion propulsion, electromagnetic pumps, MHD power generators, controlled fusion research, plasma jets and chemical synthesis, etc. Later, Attia and Kotb [4] studied the steady flow and heat transfer by considering an exponential temperature variation of the viscosity in a parallel-plates channel; injection velocity in the plates and movement of the upper plate were considered as well. Attia [5] extended the analysis in [4] including the transient term for the flow field. Analysis of the interaction of a magnetic field with other important physical effects for various situations can be found in [1, 2, 6, 7, 8]. Lima. et.,al [10] investigate Generalized Integral Transform Technique is obtained for the MHD flow and heat transfer of a Newtonian fluid in parallel-plates channels. The study of the effects of frictional heating and distributed heat sources in the fluid on fully developed laminar free convection flow between vertical heated plates, when the temperature of the walls are kept constant or varying linearly along the plate length, has been done by Ostrach [12]. Nanda and Sharma [11] have extended this to a vertical circular pipe. The corresponding problem of free convection flow between long parallel plates with porous walls has been analyzed by Rao [13]. In all these analyses a linear density-temperature variation has been taken into account which gives rise to free convection flows. Goren [9] has obtained a similarity solution. Alireza and Sahai [3] studied the effect of temperatures-dependent transport properties on the developing magneto hydrodynamic flow and heat transfer in a parallel plate channel whose walls are held at constant and equal temperatures.

In this paper we investigate laminar MHD flow in the entrance region of plane channel, under the influence of transverse magnetic field. The motion of laminar conducting fluid between two parallel plates is considered. The velocity, temperature and pressure are discussed for the different values of magnetic field parameter M and fixed prandtl number P_r.

2. FORMULATION AND SOLUTION OF THE PROBLEM

The motion of a laminar conducting fluid between two parallel plates is considered. The channel has a height of 2a and a width of d. It is of an arbitrary length, but the length must be long compared with the development length and the height. The fluids will entree the channel on the left in the figure 1. A rectangular co-ordinate system is placed on channel with the origin at the center of the channel at the left-hand edge. The variable y increases in an upward direction and the variable z increases perpendicularly to both x and y so as to form a right-handed co-ordinate system. The velocity and temperature profiles develop towards the direction of increasing x. A uniform magnetic field of strength H_0 is applied in the positive y-direction from bottom to top of the channel. The hydro magnetic equations for the governing flow are

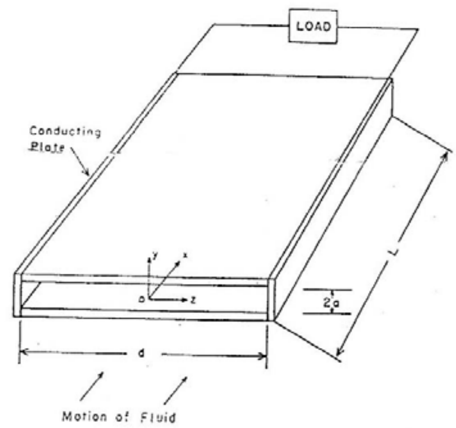


Fig.1: Channel configuration

$$\frac{D\rho}{Dt} + \rho\nabla.\overline{q} = 0 \tag{1}$$

$$\rho\frac{D\overline{q}}{Dt} = -\nabla p + \mu\nabla^2\overline{q} + \mu_e\overline{J}\times\overline{H} + \overline{F} \tag{2}$$

where $\overline{F}$ is the body force for unit volume vector

$$\rho\, C_p \frac{DT}{Dt} = \kappa \nabla^2 T + \mu\phi \tag{3}$$

where $\phi = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right]$

$\overline{J} = \sigma\left[\overline{E} + \mu_e \overline{q} \times \overline{H}\right]$, $\overline{q}$ = *(u, v, 0)* and $\overline{H}$ = *(0, H_0, 0)*

The following assumptions are made:

1. Flow is steady, laminar, viscous, incompressible and developed.

2. It is assumed that electric field $\overline{E}$ and induced magnetic field are neglected.

3. All the physical properties of the fluid are assumed to be constant.

4. It is assumed that body force is neglected.

5. Energy dissipation is neglected.

Using the above assumptions the governing equations becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = \frac{\partial^2 u}{\partial y^2} - \mu\frac{\partial p}{\partial x} - \sigma\, \mu_e^2 H_0^2 u \tag{5}$$

$$\rho C_P\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = \kappa\frac{\partial^2 T}{\partial y^2} \tag{6}$$

The boundary conditions are

$u = u_0$, $v = 0$, $T = T_{in}$ for $x = 0$ and $0 \le y \le a$ (7)

$u = 0, v = 0, T = T_w$ for $x > 0$ and $y = a$

The equations (4) to (6) and boundary conditions (7) are put in non-dimensional form by using the following transformations

$$X = \frac{x\mu}{\rho u_0 a^2},\ Y = \frac{y}{a},\ U = \frac{u}{u_0},\ V = \frac{\rho v a}{\mu},\ \theta = \frac{(T - T_{in})}{(T_W - T_{in})},\ P = \frac{P}{\rho u_0^2} \tag{8}$$

The non-dimensional governing equations are

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{9}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}-\frac{dP}{dX}-M^2U \qquad (10)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \qquad (11)$$

where

$M^2=\frac{\sigma\mu_e^2H_0^2a^2}{\mu}$ is the magnetic parameter (Hartmann number) and

$\Pr=\frac{\mu C_P}{\kappa}$ is the Prandtl number

The boundary conditions are

$$\begin{aligned} &U = 1, V = 0, \theta = 0 && \text{for } X = 0 \text{ and } 0 \le Y \le 1 \\ &V = 0 && \text{for } X > 0 \text{ and } Y = 1 \\ &U = 0, V = 0, T = 1 && \text{for } X > 0 \text{ and } Y = 1 \end{aligned} \qquad (12)$$

A finite difference technique is adopted for solving the above differential equations (9) to (11) together with boundary conditions (12).

3. RESULTS & DISCUSSIONS

We discuss the velocity, temperature and pressure distributions for different sets of governing parameters namely M, the magnetic field parameter and Pr the Prandtl number, X and Y are the variable parameters, T_w the wall temperature and T_{in} input temperature. Also for computational purpose, we are fixing the non-dimensional pressure difference P, non-dimensional stream wise velocity U as well as V non-dimensional transverse velocity and U_0 uniform velocity.

The magnitude of the velocity reduces with increase the variable parameters Y while fixing the other parameters and there is no induced magnetic field (fig.1). When we introduce the magnetic field, the similar behavior of the velocity components is observed for M= 2, 4 and 6. Next we observe that the velocity profile for Y=0.2 level with increasing the magnitude of the velocity components decreases with increase in the magnitude parameter M at Y=0.2 level and fixing the other parameters (fig. 2). When we increase the level of Y, the similar behavior of the velocity components is observed with increasing the magnetic parameter M. The magnitude of the velocity components enhances for the values of Y, 0≤Y≤0.5 and decreases for the values of 0.6≤Y≤0.9 with increase in the variable parameter X while fixing the other parameter and there is no induced magnetic field M(fig.3). From (fig.4) we notice that the temperature distribution enhances with increasing the variations of the variable parameter Y and for M=0. The similar behaviour is observed for the intensively of the magnetic field M=2, 4 and 6. We observe that the temperature distribution at various levels of Y i.e. Y=0.2, 0.6 and 0.8 enhances with increasing magnetic parameter M (M=0, 2, 4, and 6) these were shown in (fig.5). The temperature decreases with increase in the intensity of magnetic field M at 0.2 level (fig.6). When we increase the level of Y, the similar behaviour was observed with increasing the intensity of the magnetic field. Finally from (fig.7 & 8) shown that the magnitude of the pressure component reduces for all values of X ($0 \le X \le 0.002$) with increasing the intensity of the magnetic field M and fixing Y, while the magnitude of pressure decreases for $0 \le X \le 0.004$ and enhances $0.006 \le X \le 0.02$ with enhancing the magnetic field parameter M and Y is fixed .

4. CONCLUSIONS

The magnitude of the velocity reduces with increase the variable parameters Y while fixing the other parameters and there is no induced magnetic field. When we introduce the magnetic field, the similar behavior is observed. The velocity profile for Y=0.2 level with increasing the magnitude of the velocity components decreases with increase in the magnitude parameter M. When we increase the level of Y, the similar behaviour of the velocity components is observed with increasing the magnetic parameter M. The magnitude of the velocity components enhances for the values of Y, $0 \le Y \le 0.5$ and decreases for the values of $0.6 \le Y \le 0.9$ with increase in the variable parameter X while fixing the other parameters. The temperature distribution enhances with increasing the variations of the variable parameter Y and for M=0. The similar behaviour is observed for the intensively of the magnetic field M=2, 4 and 6.

The temperature distribution at various levels of Y enhances with increasing the magnetic parameter M. The temperature decreases with increase in the intensity of magnetic field M at Y=0.2 level. The magnitude of the pressure component reduces for all values of X ($0 \le X \le 0.002$) while it decreases for $0 \le X \le 0.004$ and enhances $0.006 \le X \le 0.02$ with enhancing the magnetic field parameter M and Y is fixed .

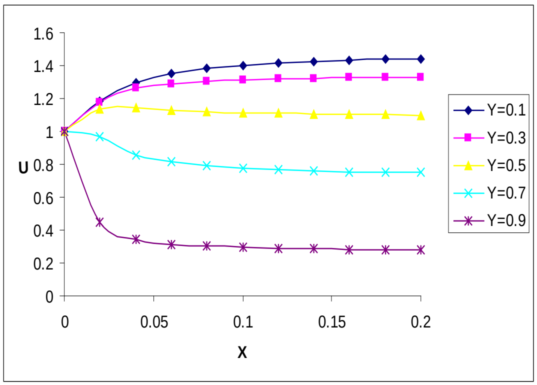


Fig 1. Velocity U for different Y with M=0

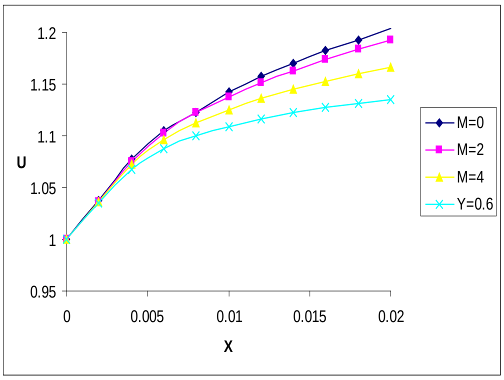


Fig2: Velocity U for different M with Y=0.2

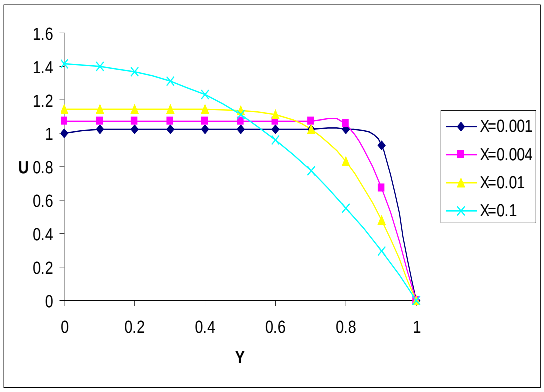


Fig.3: Velocity U for different X with M=0

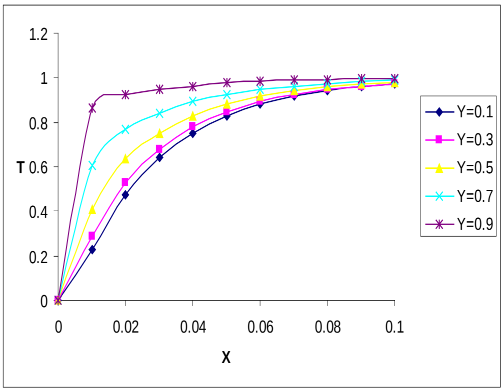


Fig.4: Temperature T for different Y with M=0

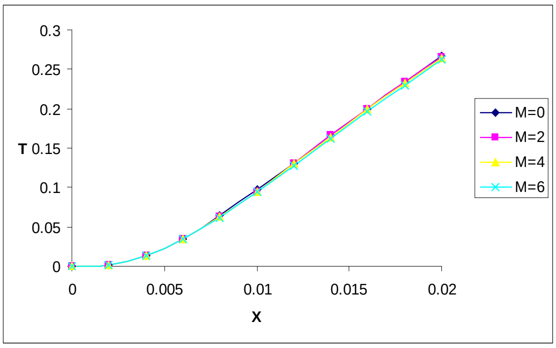


Fig.5: Temperature T for different M with Y=0.2

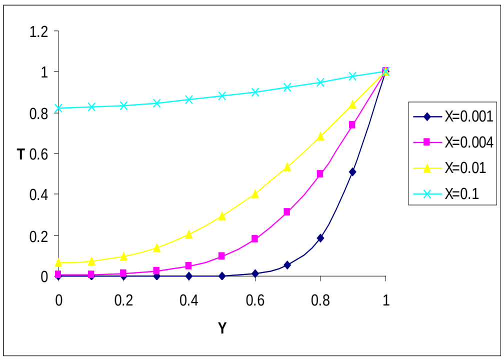
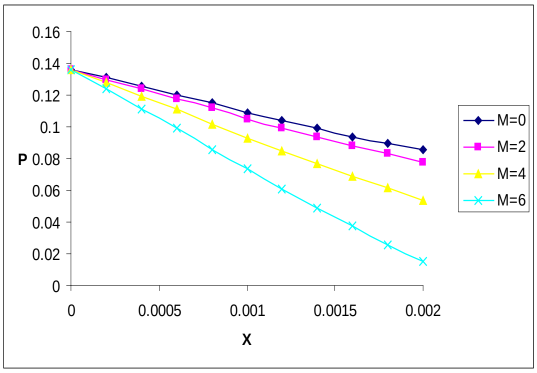


Fig.6: Temperature T for different X with M=0



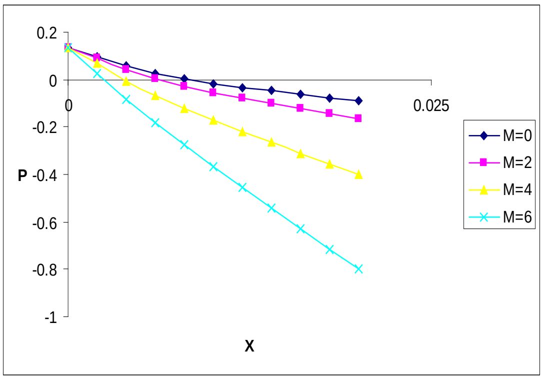


Fig. 7: Pressure P for different M Fig. 8: Pressure P for different M

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