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A NEW TECHNIQUE APPLIES FOR SOLVING FUZZY TRANSPORTATION PROBLEM USING HEXAGON NUMBER WITH ALPHA CUT RANKING TECHNIQUE

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Abstract: *Transportation Model (TP) is widely used to solve real life problems. Such as scheduling, production, investment, plant' location, inventory control, personal assignment and many others, so that this model is really not confined to transportation or distribution only: There are many technique to solve TP classically. Under many conditions exact data are inadequate to model in real life situations. Human judgments including preference are often vague and they cannot estimate preferences with an exact numerical data. Ranking fuzzy numbers is an important tool in decision making. In fuzzy decision analysis fuzzy quantities are used to describe the performance of alternatives in modeling a real world problem. In this article using Robust's Technique, to find the least transportation cost of some commodities.We introduce a new approach (method) solving TP using hexagon Number with Alpha – cut for ranking technique and given suitable numerical example*

Keywords: *Hexagon Number, Robust ranking technique, Matrix – Minima, Optimum Utilization, Fuzzy Linear Programming Problem (FLLP)*

1. INTRODUCTION

The Fuzzy Transportation Problem (FTP) is one of the special kinds of Fuzzy Linear Programming Problems (FLPP). In many fuzzy decision problems, the data are represented in terms of fuzzy numbers. The aim of FTP is to find least transportation cost of some commodities through capacitate networks. When the supply and demand of nodes and the capacity and cost of edges ate represented as fuzzy numbers.

The TP with fuzzy cost, supply and demand quantities are fuzzy quantities and the objective function is also considered as a fuzzy number. The method is to the rank the fuzzy objective values of the objective function by some ranking method for numbers to find the best alternative. On the basis of this idea the Robust Ranking Method (RRM) with help of $\alpha - cut$ solution has been adopted a transform the Fuzzy Transportation Problem.

Again this idea is to transform a problem with fuzzy parameters in the form Linear Programming Problem and solve it by the Matrix Minima method. The TP is a type of linear programming problem which deals with distribution of single product (Either raw or finished) from various sources of supply to various destination of demand in such a way that the total transportation cost is minimized.

There are effective algorithms for solving the TP with all the decision parameters. This is the supply available at each source, the demand required at each destination as well as the unit transportation costs are given in precious way.

But in real life, there are many diverse situations due to the uncertainty in one or more decision parameters and hence they may not be expressed in a precise way. This is due to measurement inaccuracy, lack of evidence, computational errors, high information cost, whether condition etc.

To deal quantitatively with imprecise information in making decision Bellman and Zadeh introduced the notion of fuzziness. In this thesis, the aim to minimize the cost, by the help of Robust Ranking Technique in TP problem with help of Hexagon Fuzzy Number concept using Matrix Reduction Method to determine the minimum cost.

2.1. Previous Study

The basic TP was originally developed by Hitchcock [6]. The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka [12] and also

Koopmans K. C [8] presented an independent study called optimum utilization of the Transportation system

These two contributions helped in the development transportation methods which involve in a number of shipping sources and a number of destinations.

Bellman and Zadeh [1] proposed the concept of decision making in fuzzy environment. After this technique introduced, many authors have studied and implemented fuzzy linear programming problem techniques such as Fang S. C., 1999 [5] Liu and Shiang [9] developed a solution procedure for computing the fuzzy objective values of the fuzzy transportation problem, where at least one the parameters are fuzzy numbers using Zadeh's extension principle in 1978.

Zimmermann. H. J. [13] has developed fuzzy linear programming in to seven fuzzy optimization methods for solving transportation problems. Iserman [7] introduced algorithm for solving transportation problem which provide effective solutions.

Different types of transportation problems were solved early by Chanas. S.; Kolodziejezyk. W.; and Machaj. A. A. [3] presented a fuzzy linear programming model for solving transportation problem with fuzzy coefficient expressed as fuzzy numbers and developed an algorithm for obtaining the optimal solution. Ringuest. J. L; and Rings. D. B. [11] proposed two iterative algorithms for solving linear multi criteria transportation problem. Campos. L and Verdegray. J [2] proposed a method for solving fuzzy transportation problem and also, to find the probability distribution of the objective value of the transportation problem provided all the inequality constraints are of "Less than or equal to" types or "Greater than or equal to" types.

Chanas. S and Kutchtain. D [4] solved fuzzy integer transportation problem and also they proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy number and developed an algorithm for obtaining the optimal solution. Pandian. P and Nagarajan. G. [10] proposed a new algorithm namely zero point method for finding optimal solution for a fuzzy transportation, the transportation cost, supply and demand are represented by trapezoidal fuzzy numbers.

2.2. Preliminaries

2.2.1. Definition for Fuzzy Set – Membership function

A fuzzy set A it is characterized by a membership function, mapping elements of a domain, space or universe of discourse X to the unit closed interval $[0, 1]$. That is,

$A = \{\langle x, \mu_A(x)\rangle | x \in X\}$, here $\mu_A : X \longrightarrow [0, 1]$, it is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ it is called the membership value of $x \in X$, in the fuzzy set A. These membership grades are often represented by real numbers ranging from $[0, 1]$

2.2.2. Definition for Normal Fuzzy Set

A fuzzy set A of the universe of discourse X it is called a normal fuzzy set if there exist at least one $x \in X$ such that $\mu_A(x) = 1$

2.2.3. Definition for Interval Number

Let $\mathbb{R}$, it is the set of real numbers and then a closed interval $[a, b]$, it is said to be an interval number, where $a, b \in \mathbb{R}$ with $a \leq b$

2.2.4. Convex normalized fuzzy set

A fuzzy number μ, it is a convex normalized fuzzy set of the crisp set such that for only one $x \in X\ amd\ \mu_A(x) = 1\ and\ \mu_A(x)$, it is piecewise continuous

2.2.5. Support of Fuzzy number

Let$u$, it is a fuzzy number and then the support of$u$, it is defined by

$\sup(u) = \overline{\{x|u(x) > 0\}}$, where$\overline{\{x|u(x) > 0\}}$, represents the closure of$\{x|u(x) > 0\}$

2.2.6. Fuzzy Number Concepts

A fuzzy set A, of the real line $\mathbb{R}$, with membership function$\mu_A: \mathbb{R} \longrightarrow [0,1]$, it is called fuzzy number if

a. A, it must be convex and normal fuzzy set
b. The support of A, it must be bounded
c. α_A, it must be closed interval for every $\alpha \in [0,1]$

2.2.7. Definition for Hexagon Fuzzy Number

A fuzzy number $A = (a, b, c, d, e, f)$, it is said to be Hexagon Fuzzy Number if its membership function $\mu_A(x)$, it is given by

$$\mu_A(x) = \begin{cases} \frac{y-a}{b-a}, & if\ a \le y \le b \\ 1, & if\ b \le y \le c \\ \frac{d-y}{d-c}, & if\ c \le y \le d \\ 0, & Otherwise \\ 1, & if\ d \le y \le e \\ \frac{f-y}{f-e}, & if\ e \le y \le f \end{cases}$$

2.2.8. Graphical representation of Hexagon Fuzzy Number

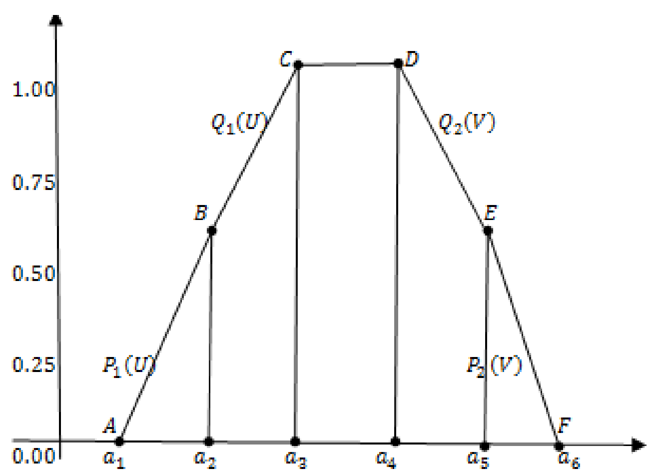

Figure – 1: Graphical Representation of Hexagonal Fuzzy Number

2.2.9. Arithmetic Operations on Hexagonal Fuzzy Numbers

Let us take two hexagonal Fuzzy numbers are

$A_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ *and* $B_H = (b_1, b_2, b_3, b_4, b_5, b_6)$, and then

a. $A_H + B_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5,\ a_6 + b_6)$
b. $A_H - B_H = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5,\ a_6 - b_6)$
c. $A_H \times B_H = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4, a_5 \times b_5,\ a_6 \times b_6)$

2.2.10. Robust Ranking Technique

Robust ranking technique which satisfies compensation, linearity, and additivity properties provides results which consist with human intuition. Suppose that a given convex fuzzy number$\tilde{a}$, the Robust Ranking Index is defined by

$R(\tilde{a}) = \int_0^1 0.5(a_{H\alpha}^L, a_{H\alpha}^U)d\alpha\ where\ (a_{H\alpha}^L, a_{H\alpha}^U)$, it is a $\alpha - level$ cut of a fuzzy number$\tilde{a}$In this article we use this method for the ranking of the objective values. The Robust ranking index $R(\tilde{a})$ gives the representative value of the fuzzy number$\tilde{a}$

2.3. Numerical Example

Consider the Fuzzy Transportation Problem that is a company has six sources they areS_1, S_2, S_3, S_4, S_5 *and* S_6, and six destinations, they areD_1, D_2, D_3, D_4, D_5 *and* D_6, and the fuzzy transportation cost for unit quantity of the product from $ith\ source\ and\ jth\ desitination$, it is given by

$$[b_{ij}]_{3\times4} = \begin{bmatrix} (1,2,3,4,5,6) & (1,3,4,6,7,8) & (8,9,7,6,5,4) & (2,6,5,4,3,2) \\ (3,6,5,4,3,2) & (2,3,5,6,7,5) & (4,7,6,5,2,1) & (3,4,5,6,7,5) \\ (1,5,6,7,6,2) & (1,8,7,6,5,6) & (5,9,4,6,7,6) & (8,7,1,0,6,5) \end{bmatrix}$$

And fuzzy availability of the products at sources they are

$(8,10,12,12,6,4); (2,3,5,6,2,1);\ and\ \ (5,10,12,17,11,10);$

And the fuzzy demand of the product at destinations, they are

$(5,8,8,7,5,4);\ (5,1,6,7,5,2);\ (2,3,1,3,5,7)\ and\ (3,6,9,12,15,18)$

Solution:

First we prepare in tabular form that is shown below:

Table – 1

	FuzzyD_1	FuzzyD_2	FuzzyD_3	FuzzyD_4	Supply
FuzzyS_1	$(1,2,3,4,5,6)$	$(1,3,4,6,7,8)$	$(8,9,7,6,5,4)$	$(2,6,5,4,3,2)$	$(8,10,12,12,6,4)$
FuzzyS_2	$(3,6,5,4,3,2)$	$(2,3,5,6,7,5)$	$(4,7,6,5,2,1)$	$(3,4,5,6,7,5)$	$(2,3,5,6,2,1)$
FuzzyS_3	$(1,5,6,7,6,2)$	$(1,8,7,6,5,6)$	$(5,9,4,6,7,6)$	$(8,7,1,0,6,5)$	$(5,10,12,17,11,10)$
Demand	$(5,8,8,7,5,4)$	$(5,1,6,7,5,2)$	$(2,3,1,3,5,7)$	$(3,6,9,12,15,18)$	

Calculation:

The fuzzy transportation problem can be formulated in the following mathematical programming as

$$\text{Min }(X) = \begin{bmatrix} R(1,2,3,4,5,6) + & R(1,3,4,6,7,8) + & R(8,9,7,6,5,4) + & R(2,6,5,4,3,2) \\ R(3,6,5,4,3,2) + & R(2,3,5,6,7,5) + & R(4,7,6,5,2,1) + & R(3,4,5,6,7,5) \\ R(1,5,6,7,6,2) + & R(1,8,7,6,5,6) + & R(5,9,4,6,7,6) + & R(8,7,1,0,6,5) \end{bmatrix}$$

And also we have

$$R(\tilde{a}) = \int_0^1 0.5(a_{H\alpha}^L, a_{H\alpha}^U)d\alpha\,, where$$

$$(a_{H\alpha}^L, a_{H\alpha}^U) = [(b-a)\alpha + a, d-(d-c)\alpha] - [(d-c)\alpha + c, f-(f-e)\alpha]$$

For example:

$$In\ R(1,2,3,4,5,6), we\ have\ a = 1, b = 2, c = 3, d = 4, e = 5\ and\ f = 6$$

$$\Longrightarrow R(1,2,3,4,5,6) = \int_0^1 (0.5)(\alpha + 1 + 4 - \alpha + \alpha + 3 + 6 - \alpha)d\alpha = \int_0^1 (0.5)(14)d\alpha$$

$$\Longrightarrow R(1,2,3,4,5,6) = \frac{1}{2}[14\alpha]_0^1 = \frac{1}{2} \times \{14 - 0\} = 7$$

Similarly we can calculate

$R(1,3,4,6,7,8) = 9.75$

$R(8,9,7,6,5,4) = 13$

$R(2,6,5,4,3,2) = 7.75$

$R(3,6,5,4,3,2) = 8$

$R(2,3,5,6,7,5) = 9.75$

$R(4,7,6,5,2,1) = 9$

$R(3,4,5,6,7,5) = 10.25$

$R(1,8,7,6,5,6) = 10$

$R(1,8,7,6,5,6) = 11.5$

$R(5,9,4,6,7,6) = 11.75$

$R(8,7,1,0,6,5) = 7$

Now Rank of all Supply

$R(8,10,12,12,6,4) = 19$

$R(2,3,5,6,2,1) = 7.5$

$R(5,10,12,17,11,10) = 23.5$

Now Rank of all Demands

$R(5,8,8,7,5,4) = 13$

$R(5,1,6,7,5,2) = 9.75$

$R(2,3,1,3,5,7) = 6.25$

$R(3,6,9,12,15,18) = 21$

Prepare Second table after ranking we have

Table – 2 - After Ranking

	FuzzyD_1	FuzzyD_2	FuzzyD_3	FuzzyD_4	Supply
FuzzyS_1	7	9.75	13	7.75	19
FuzzyS_2	8	9.75	9	10.25	7.5
FuzzyS_3	10	11.5	11.25	7	23.5
Demand	13	9.75	6.25	21	

2.3.1. Matrix Minima Method

This method consists in allocating as much as possible in the lowest cost cell/cells and then further allocation is done in the cell/cells with second lowest cost and so on. In case of tie among the cost select the cell where allocation of more number of units can be made

Continue our Problem by applying matrix minima method

That is in the table – 2, here, the lowest cost cell is$(1,1)$, and maximum possible allocation (Need supply and demand positions) is made here

Evidently the maximum feasible allocation in cell$(2,3)$, it is 9 this meets the supply position of demand 2. Therefore, row 2 is crossed out, indicating that no allocation are to made in cell$(2,1),(2,2)$ *and* $(2,4)$

The next lowest cost cells are$(3,4)$, , and also second lowest cost is in the cell$(3,3)$

So that we get lowest cost cells are$(1,1)$, $(2,3),(3,3)$ *and* $(3,4)$

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Table – 3 – After applying matrix minima method

	FuzzyD_1	FuzzyD_2	FuzzyD_3	FuzzyD_4	Supply
FuzzyS_1	**7**(minimum in Row and Column)				
	7	~~9.75~~	~~13~~	~~7.75~~	19
FuzzyS_2	**9**(minimum in Row and Column)				
	8	9.75	9	~~10.25~~	7.5
FuzzyS_3			Take both		
	10	11.5	$\mathbf{11.25}$	$\mathbf{7}$	23.5
Demand	13	9.75	6.25	21	

The transportation cost associated with this solution is

$$Z_{min} = (13)(7) + (9.75)(9) + (11.25)(6.25) + (7)(21)$$

$$\Longrightarrow Z_{min} = 91 + 87.75 + 70.3125 + 147 = 396.0625$$

2.4. Conclusion

We discussed in this article solving fuzzy transportation problem by new approach and we solve suitable numerical example also in this article.

From this result of numerical example, we conclude that optimal solution for a fuzzy transportation problem using hexagon fuzzy numbers by using robust technique, we will get better result compare to other previous techniques. This latest technique is called Robust Technique implemented in Fuzzy hexagon numbers Computational procedure to find better optimal solution. This approach of solving fuzzy problems may also be utilized in further studies of operation research.

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