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MULTIPLE ATTRIBUTE GROUP DECISION MAKING PROBLEMS BY USING OPERATOR THE INDUCED VAGUE ORDERED WEIGHTED AVERAGING (IVOWA) APPEAR FOR VAGUE SETS WITH NEW DEVELOPED ALGORITHM

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Abstract: *There are several approaches to extend the basic multi attribute decision making techniques for the case of group decision. If the objective function is the single criterion, the constraints are the requirements on the alternatives. Depending on the form and functional description of the optimization problem like as linear, non – linear programming, discrete optimization techniques etc. But it is very important to make a decision between the cases where we have a single or multiple criteria. When a decision problem has a finite number of criteria or multiple criteria, and the number of the feasible alternatives (the one meeting requirements) is infinite, then the decision problem belongs to the field of multiple criteria optimization. Also, technique of multiple criteria optimization can be used when there are a finite number of feasible alternatives, but are given only in implicit form. In this article we focus on making problem when the number of the criteria (attribute) and alternatives is finite and the alternatives are explicitly given. And also the ordered weighted average is presented in the context of vague set theory. Vague sets can better handle vagueness and uncertainty than intuitionistic fuzzy sets. Initially the vague weighted averaging (VWA) operator was developed and based on VWA operator the vague ordered weighted averaging (VOWA) operator was introduced. Finally the induced ordered weighted averaging (IVOWA) operator was developed and MAGDM model was proposed using all the above proposed operators. The proposed model of MADGM can be utilized based on distance function by using Hausdorff Metric distance technique*

Keywords: *Hausdorff Metric Distance, Discrete Optimum Technique, Intuitionistic Fuzzy Sets, Distance between two intuitionistic fuzzy sets.*

1. INTRODUCTION

A general comprehensive methodology for a wide class MAGDM models is using aggregation based on generalized means, including the additive and multiplicative models as well. In the approach the weights and the scores of the alternative against the criteria can change simultaneously in given interval. The following questions are to be added.

- What are the intervals of the final ranking values of the alternatives with the restriction that the intervals of the weight and scores are given?
- What are the intervals of the weights and scores with the restriction that the final ranking of the alternatives does not change?
- Consider a subset of alternative whose ranking values are allowed to change in the interval. In what intervals are the weights and scores allowed to vary, and how will these modifications affect the ranking values of the entire set of alternatives?

Sensitive analysis is done based on these above mentioned questions.

2. PREVIOUS WORK

Since the theory of fuzzy sets was proposed in 1965 by Zadeh, [9] it has been applied in many uncertain information processing problems successfully, since in the real world there is vague information about different applications. Gau&Buehrer [1994] [3] pointed out the drawback of using the single membership value in fuzzy set theory. In order to tackle this problem, they proposed the notion of vague sets (VSs). In the chapter – I, in introduction part the notion of IFSs and VSs are regarded as equivalent, in the sense of isomorphic to VS.

3. PRELIMINARIES

3.1. Intuitionistic Fuzzy Sets and Interval – Valued Fuzzy Sets

Let X denotes a universe of discourse. Then a fuzzy set $A\ in\ X$, it is defined as a set of ordered pairs $A = \{\langle x, \mu_A(x)\rangle | x \in X, \}, where\ \mu_A: X \longrightarrow [0,1]$ it is a membership function of $A\ and\ \mu_A(x)$, it is the grade of belongingness of $x\ into\ A$

Thus, automatically the grade of non – belongingness of $x\ into\ A$, it is equal to

$1 - \mu_A(x)$. However, in real life the linguistic negation not always identifies with logical negation. This situation is very common in naturallanguage processing, computing with words, etc. Therefore Atanassov [1986] [1] suggested a generalization of classical fuzzy set, called an intuitionistic fuzzy set.

3.2. Definition for Intuitionistic Fuzzy Set

An intuitionistic fuzzy set $A\ in\ X$, it is given by a set of ordered triples

$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X, \}, where\ \mu_A, v_A: X \longrightarrow [0,1]$, they are functions such that

$0 \le \mu_A(x) + v_A(x) \le 1, \forall\ x \in X$. For each x, the numbers$\mu_A(x)\ and\ v_A(x)$ represent the degree of membership and degree of non – membership of the element $x \in X\ to\ A \subset X$

It is easily to see that$\{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in X, \}$, it is equivalent to 1, i.e. each fuzzy set is a particular case of the intuitionistic fuzzy set. A family of fuzzy sets in X, it is denoted by $FS(X)$, while $IFS(X)$ stands for the family of all intuitionistic fuzzy sets in X

For each element $x \in X$, the intuitionistic fuzzy index of $x\ in\ A$ defined as follows:

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x)$$

Then$\pi_A(x) \in [0,1], \forall\ x \in X$, and if $A \in\ FS(X), then\ \pi_A(x) = 0, \forall\ x \in X$

3.3. Definition for Interval – Valued fuzzy set (IVF)

Some notation is used in the interval – valued fuzzy set theory.

Let$[I] = [a^-, a^+], for\ a^- \le a^+\ with\ a^-, a^+ \in [0,1]$, and then the mapping

$A: X \longrightarrow [I]$, it is called an interval – valued fuzzy seton X

Let $IVF(X)$, denote all interval –valued fuzzy sets on X. Foreach$A \in IVF(X)$,

Let $A(x) = [A^-(x), A^+(x)]\ where\ A^-(x) \le\ A^+(x)\ and\ x \in X$

Then the fuzzy set$A^-(x): X \longrightarrow [I]\ and\ A^+(x): X \longrightarrow [I]$, they are called a lower fuzzy set of A, and an upper fuzzy set of A

3.4. Four Measuring distances between fuzzy sets

In many theoretical and practical problems, it is to express numerically the difference between two objects (notion, etc) by means of a distance between corresponding fuzzy sets. From here we will assume that the universe of discourse under study is finite, i.e. $X = \{x_1, \dots, x_n\}$. One can define and use different metrics in a family of fuzzy subsets of given universe of discourse X: Especially, the Hamming metric and the Euclidean metric are most often used.

For any two fuzzy subsets $A\ and\ B\ of\ X$, with membership functions$\mu_A\ and\ \mu_B$, respectively, it follows that

a. The Hamming Distance $d(A, B) = \sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|$

b. The Normalized Hamming Distance $l(A, B) = \frac{1}{n}\sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|$

c. The Euclidean Distance $e(A,B) = \sqrt{\sum_{i=1}^{n}\left(\mu_A(x_i) - \mu_B(x_i)\right)^2}$

d. The Normalized Euclidean Distance $q(A,B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\mu_A(x_i) - \mu_B(x_i)\right)^2}$

3.5. Four Measuring between Intuitionistic Fuzzy Sets

These formulas are straightforward generalizations of distance used in classical set theory obtained by replacing the characteristic functions of two sets with the membership functions. Atanassov [1986] [1] suggested a direct generalization of distance for intuitionistic fuzzy sets:

For $A, B \in IFS(X)$, it gets that

a. The Hamming Distance

$$d'(A,B) = \frac{1}{2}\sum_{i=1}^{n}\{|\mu_A(x_i) - \mu_B(x_i)| + +|v_A(x_i) - v_B(x_i)|\}$$

b. The Normalized Hamming Distance

$$l'(A,B) = \frac{1}{2n}\sum_{i=1}^{n}\{|\mu_A(x_i) - \mu_B(x_i)| + +|v_A(x_i) - v_B(x_i)|\}$$

c. The Euclidean Distance

$$e'(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left\{\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(v_A(x_i) - v_B(x_i)\right)^2\right\}}$$

d. The Normalized Euclidean Distance

$$q'(A,B) = \sqrt{\sum_{i=1}^{n}\frac{1}{2n}\left\{\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(v_A(x_i) - v_B(x_i)\right)^2\right\}}$$

3.6. Four Measuring Distance between Fuzzy Sets using Fuzzy Index

Starting from the geometrical interpretation of intuitionistic fuzzy sets Szmidt&Kacpryzk [2000] [6] modified these distances. They proposed to take into account the three parameters characterization of intuitionistic fuzzy sets. The degree of membership $\mu(x)$, and the degree of non – membership $v(x)$ and the intuitionistic fuzzy index $\pi(x)$: Here the definitions of distances are given by Szmidt & Kacpryzk [2000] [6]

a. The Hamming Distance

$$d^{''}(A,B) = \frac{1}{2}\sum_{i=1}^{n}\{|\mu_A(x_i) - \mu_B(x_i)| + +|v_A(x_i) - v_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|\}$$

b. The Normalized Hamming Distance

$$l^{''}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}\{|\mu_A(x_i) - \mu_B(x_i)| + +|v_A(x_i) - v_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|\}$$

c. The Euclidean Distance

$$e^{''}(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left\{\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(v_A(x_i) - v_B(x_i)\right)^2 + |\pi_A(x_i) - \pi_B(x_i)|^2\right\}}$$

d. The Normalized Euclidean Distance

$$q^{'}(A,B) = \sqrt{\sum_{i=1}^{n}\frac{1}{2n}\left\{\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(v_A(x_i) - v_B(x_i)\right)^2 + |\pi_A(x_i) - \pi_B(x_i)|^2\right\}}$$

3.7. Distance between on the Hausdorff Metric

Szmidt&Kacpryzk [2000] [6] claim that approaches ensure that the distance for fuzzy sets and intuitionistic fuzzy sets can be easily since it reflects distances in three dimensional spaces, while distances due to Atanassov[1] are the orthogonal projection of the real distances. Nevertheless this reasoning seems to be somehow strange and non – convincing since their modification reduces to add the parameter that is a linear combination of two other parameters used in the Atanassov [1986, 1989] [1,2] definitions

3.8. Definition for Hausdorff Metric

Besides distances described in the previous section, some distance based on the Hausdorff metric are also used in the fuzzy sets theory.

For any two subsets $U \; and \; W$ of a Banach Space Z, the Hausdorff Metric is

$$d_H(U,W) = \max\left\{\sup_{u\in U}\inf_{u\in U}|u-w|, \sup_{u\in U}\inf_{u\in U}|u-w|\right\}$$

If $Z = \mathbb{R}, U = [u_1, u_2] \; and \; W = [w_1, w_2]$, they are intervals and then reduces to

$$d_H(U,W) = \max\{|u_1 - w_1|, |u_2 - w_2|\}$$

It seems that this metric applied for intervals could be successfully used in the case of intuitionistic fuzzy sets too. Let us start from the following example connected with a decision making problem.

3.9. Note

As it was shown in the example there is one – to – one correspondence between interval valued intuitionistic fuzzy sets and the intuitionistic fuzzy set description based on intervals of possible values that the degree of membership and degree of non – membership of each element could assumed if the hesitation were removed. And hence, one can look on the distance between two intuitionistic fuzzy sets A and B in the degenerated universe of discourse

$X = \{x\}, where\ A = \{\langle x, \mu_A(x), v_A(x)\rangle\}\ and\ B = \{\langle x, \mu_B(x), v_B(x)\rangle\}$, respectively, as on the distance between corresponding intervals

$$[\mu_A(x), 1 - v_A(x)]\ and\ [\mu_B(x), 1 - v_B(x)]$$

Using the Hausdorff Metric, we get

$$d(A, B) = \max\{|\mu_A(x) - \mu_B(x)|, \left|\left(1 - v_A(x)\right) - \left(1 - v_B(x)\right)\right|\}$$

$$\Longrightarrow d(A, B) = \max\{|\mu_A(x) - \mu_B(x)|, |v_A(x) - v\mu_B(x)|\}$$

3.10. Example

Consider the following intuitionistic fuzzy sets $A, B, D, G, E \in X = \{x\}$, where

$$A = \{\langle x, 1, 0\rangle\}; B = \{\langle x, 0, 1\rangle\}; D = \{\langle x, 0, 0\rangle\}; G = \left\{\langle x, \frac{1}{2}, \frac{1}{2}\rangle\right\}\ and\ E = \left\{\langle x, \frac{1}{4}, \frac{1}{4}\rangle\right\}$$

Going back to the decision making problem discussed above we can say that

Expert A, it is completely convinced to vote for

Expert B, it is completely to vote against for

Expert C, it is absolutely hesitate (that is undecided whether vote for or against)

According to definitions we have

$$d(A, B) = \max\{|1 - 0|, |0 - 1|\} = 1$$

$$d(A, D) = \max\{|1 - 0|, |0 - 0|\} = 1$$

$$d(B, D) = \max\{|0 - 0|, |1 - 0|\} = 1$$

$$d(A, G) = \max\left\{\left|1 - \frac{1}{2}\right|, \left|0 - \frac{1}{2}\right|\right\} = \frac{1}{2}$$

$$d(A, E) = \max\left\{\left|1 - \frac{1}{4}\right|, \left|0 - \frac{1}{4}\right|\right\} = \frac{3}{4}$$

$$d(B, G) = \max\left\{\left|0 - \frac{1}{2}\right|, \left|1 - \frac{1}{2}\right|\right\} = \frac{1}{2}$$

$$d(B,E) = \max\left\{\left|0-\frac{1}{4}\right|, \left|1-\frac{1}{4}\right|\right\} = \frac{3}{4}$$

$$d(D,G) = \max\left\{\left|0-\frac{1}{2}\right|, \left|0-\frac{1}{2}\right|\right\} = \frac{1}{2}$$

$$d(D,E) = \max\left\{\left|0-\frac{1}{4}\right|, \left|0-\frac{1}{4}\right|\right\} = \frac{1}{4}$$

$$d(G,E) = \max\left\{\left|\frac{1}{2}-\frac{1}{4}\right|, \left|\frac{1}{2}-\frac{1}{4}\right|\right\} = \frac{1}{4}$$

Now it is able to suggest how to measure the distance between fuzzy sets on arbitrary finite universe of discourse the Hausdorff metric. Our definitions are natural counterparts of the Hamming distance the Euclidean distance and their normalized versions

3.11. Definition

For any two intuitionistic fuzzy subsets

$A = \{\langle(x_i), \mu_A(x_i), v_A(x_i)|x_i \in X\rangle\}$ *and* $B = \{\langle(x_i), \mu_B(x_i), v_B(x_i)|x_i \in X\rangle\}$, of the universe of discourse $X = \{x_1, \ldots, x_n\}$, it follows that

a. The Hamming Distance

$$d_h(A,B) = \sum_{i=1}^{n} \max\{|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|\}$$

b. The Normalized Hamming Distance

$$l_h(A,B) = \frac{1}{n}\sum_{i=1}^{n} \max\{|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|\}$$

c. The Euclidean Distance

$$e_h(A,B) = \sqrt{\sum_{i=1}^{n} \max\left\{\left(\mu_A(x_i) - \mu_B(x_i)\right)^2, \left(v_A(x_i) - v_B(x_i)\right)^2\right\}}$$

d. The Normalized Euclidean Distance

$$q_h(A,B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \max\left\{\left(\mu_A(x_i) - \mu_B(x_i)\right)^2, \left(v_A(x_i) - v_B(x_i)\right)^2\right\}}$$

Based on the above discussion, let us define a new distance function between two vague values

3.12. Definition

Let $A = \left(t_A(x_i), f_A(x_i)\right)$ and $B = \left(t_B(x_i), f_B(x_i)\right)$, they are two vague values. Then the Euclidean distance between A and B, it is given as follows:

$$d(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left\{\left(t_A(x_i) - t_B(x_i)\right)^2 + \left(\left(1 - f_A(x_i)\right) - \left(1 - f_B(x_i)\right)\right)^2\right\}}$$

3.13. The IVOW operator for Group Decision Making

MAGDM with IFS (Intuitionistic Fuzzy Sets) Theory

Multi Attribute Group Decision Making (MAGDM) problems are wide spread in real life situation. A MAGDM problem is to find a desirable solution from a finite number of feasible alternatives assessed on multiple attributes, both quantitative and qualitative. To choose a desirable solution, the decision maker often provides his/her preferred information in the form of numerical values, such as exact values, interval member values and fuzzy numbers.

However, under many conditions, numerical values are inadequate or insufficient to model real life decision problems. Indeed, human judgments including preference information may be stated in intuitionistic fuzzy information. Hence MAGDM problems under intuitionistic fuzzy environment are an interesting are of study for researchers recently.

Processing of MAGDM problems with intuitionistic fuzzy information or vague fuzzy information, sometimes, leads to attribute values taking the form of intuitionistic or vague fuzzy number, respectively. The information about attribute weights may sometimes be known, partially known or be completely unknown. MAGDM problems are assumed to have a predetermined, limited number of decision alternatives

Solving a MAGDM problem involves sorting and ranking, and can be viewed as an alternative method for combining information in problem's decision matrix together with additional information from the decision maker to determine a final ranking or selection from the alternatives. Besides the information contained in the decision matrix, all but the simplest MAGDM techniques require additional information from the decision matrix to arrive at a final ranking/selection

Li [2005] [4] presented a new method for handling multiple attribute fuzzy decision making problems, where the characteristics of the alternatives are represented by intuitionistic fuzzy sets. Xu and Yager [2006] [8] developed some geometric aggregation operators for MAGDM problems. Liu and Wang [2009] [5] developed an evaluation function the decision making problems to measure the degrees to which alternatives satisfy/do not satisfy the decision maker's requirements. Wei and Zhao [2012] [7] contributed novel approaches to the field of fuzzy decision making

In this article, a new operator is defined the induced vague ordered weighted averaging (IVOWA) appear for vague sets

3.14. Definition for VWA Operator

Let$a_j = (t_i, f_j), for\ j = 1, \dots, n$, it is a collection of vague values, and let the vague weighted averaging operator VWA is defined as:

$$VWA: Q'' \longrightarrow Q$$

$$if\ VWA_\omega(a_1, \dots, a_n) = \sum_{j=1}^{n} w_j a_j = \left[1 - \prod_{j=1}^{n}(1 - t_j)^{\omega_j}, \prod_{j=1}^{n}(1 - f_j)^{\omega_j}\right]$$

$Where\ \ \omega = (\omega_1, \dots, \omega_n)^T$, it is the weight vector of

$$a_j = (t_j, f_j); and\ \omega_j > 0, \sum_{j=1}^{n} \omega_1 = 1$$

3.15. Definition for VOWA Operator

Let$a_j = (t_j, f_j), for\ j = 1, \dots, n$, it is a collection of vague values, and let the vague ordered weighted averaging operator VOWA of dimension n, it is given by the mapping

$VOWA: Q^n \longrightarrow Q$, with an associated weight vector $\omega = (\omega_1, \dots, \omega_n)^T$, such that

$$\omega_j > 0, \sum_{j=1}^{n} \omega_1 = 1\ \ then$$

$$VOWA_\omega(a_1, \dots, a_n) = \sum_{j=1}^{n} w_j a_{\sigma(j)} = \left[1 - \prod_{j=1}^{n}(1 - t_{\sigma(j)})^{\omega_j}, \prod_{j=1}^{n}(1 - f_{\sigma(j)})^{\omega_j}\right]$$

Where$(\sigma(1), \sigma(2), \dots, \sigma(n))$, it is a permutation of$(1,2, \dots, n)$ $such\ that$

$$a_{\sigma(j-1)} \geq a_{\sigma(j)}\ for\ all\ j = 2, 3, \dots, n$$

3.16. Definition for IVOWA Operator

TheIVOWA Operator is defined as follows:

$$IVOWA_\omega(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) = \sum_{j=1}^{n} w_j g_j = \left[1 - \prod_{j=1}^{n}(1 - \bar{t}_j)^{\omega_j}, \prod_{j=1}^{n}\left(1 - \overline{f}_j\right)^{\omega_j}\right]$$

$Where\ \ \omega = (\omega_1, \ldots, \omega_n)^T$, it is the weight vector such that $\omega_j \in [0, 1]$, and $\sum_{j=1}^{n} \omega_1 = 1$

$g_j = \left(\bar{t}_j, \bar{f}_j\right)$, it is the value of VOWA pair $\langle u_i, a_i \rangle$, it is have the jth largest $u_i \in [0, 1]$, and in $\langle u_i, a_i \rangle$, and it is called the order inducing variable and a_i, it is the vague value. The IVOWA operator satisfies the following properties:

3.17. IVOWA Operator is Commutative

$$IVOWA_\omega(\langle u_1, a_1 \rangle, \ldots, \langle u_n, a_n \rangle) = IVOWA_\omega(\langle u_1, a_1^{'} \rangle, \ldots, \langle u_n, a_n^{'} \rangle)$$

$Where\ (\langle u_1, a_1^{'} \rangle, \ldots, \langle u_n, a_n^{'} \rangle)$, it is any permutation of $(\langle u_1, a_1 \rangle, \ldots, \langle u_n, a_n \rangle)$

3.18. IVOWA Operator is Idempotency

If $a_j = a, where\ \ a_j = \left(t_j, f_j\right)\ and\ a = (t, f)\ for\ all\ j$, and then

$$IVOWA_\omega(\langle u_1, a_1 \rangle, \ldots, \langle u_n, a_n \rangle) = a$$

3.19. IVOWA Operator is Monotonicity

If $a_j \le a_j^{'}\ \ for\ all\ j$, and then

$$IVOWA_\omega(\langle u_1, a_1 \rangle, \ldots, \langle u_n, a_n \rangle) \le IVOWA_\omega(\langle u_1, a_1^{'} \rangle, \ldots, \langle u_n, a_n^{'} \rangle)$$

3.20. Proposed Model of MAGDM

Let $A = \{A_1, A_2, \ldots, A_n\}$, it is the set of alternatives and $G = \{G_1, G_2, \ldots, G_n\}$, it is the set of attributes and $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$, it is the weighting vector of the attributes

$G_j, j = 1, \ldots, n\ where\ \ \omega_j \in [0, 1]$, and $\sum_{j=1}^{n} \omega_1 = 1$

Let $D = \{D_1, D_2, \ldots, D_n\}$, it is the set of decision makers and $V = \{V_1, V_2, \ldots, V_n\}$, it is the weighting vector of the decision makers with $V_k \in [0, 1]\ and\ \sum_{k=1}^{n} V_k = 1$

Let $R_k = \left(\bar{r}_{ij}^{(k)}\right)_{m \times n} = \left(t_{ij}^{(k)}, f_{ij}^{(k)}\right)_{m \times n}$, it is the vague decision matrix where

$t_{ij}^{(k)}$, it is the degree of the truth membership value that the alternative A_i satisfies the attributes G_j and it is given by the decision maker $D_k\ and\ f_{ij}^{(k)}$, it is the degree of false membership value that the alternative for the alternative A_i, where $t_{ij}^{(k)}, f_{ij}^{(k)} \in [0, 1]$, and

$$t_{ij}^{(k)} + f_{ij}^{(k)} \le 1, for\ i = 1, 2, \ldots, m, j = 1, 2, \ldots, n\ and\ k = 1, 2, \ldots, t$$

3.21. An Algorithm for a Developed Model of MAGDM

The following steps are now given:

Step – 1:

Utilize the vague decision matrix$R_k = \left(\overline{r}_{ij}^{(k)}\right)_{m\times n} = \left(t_{ij}^{(k)}, f_{ij}^{(k)}\right)_{m\times n}$, and the IVOWA operator which has the associated weighting vector $\omega = (\omega_1, \dots, \omega_n)^T$, and also

$$\overline{r}_{ij}^{(k)} = \left(t_{ij}^{(k)}, f_{ij}^{(k)}\right) = IVOWA_\omega\left(\langle V_1, \overline{r}_{ij}^{(1)}\rangle, \dots, \langle V_t, \overline{r}_{ij}^{(t)}\rangle\right)$$

$for\ i = 1, \dots, m\ and\ j = 1, \dots, n$, to aggregate into a collective decision matrix

$R_k = \left(\overline{r}_{ij}^{(k)}\right)_{m\times n}$ $where\ V = (V_1, V_2, \dots, V_t)^T$, it is the weighting vector of decision maker.

Step – 2:

Utilizing the information from the collective decision matrix $R_k = \left(\overline{r}_{ij}^{(k)}\right)_{m\times n}$, and the VWA operator$\overline{r}_i = (t_i, f_i) = VWA_\omega = (\overline{r}_{i1}, \overline{r}_{i2}, \dots, \overline{r}_{in}), for\ i = 1, \dots, m,$ derive the overall performance values of the alternatives$A_i\ when\ \omega = (\omega_1, \dots, \omega_n)^T$, it is the weighting vector of the attributes.

Step – 3:

Calculate the distance between the collective overall performance values and the positive ideal vague value$\overline{r}^+$, or the negative ideal vague value$\overline{r}^-$,

$$where\ \overline{r}^+ = (1, 0)\ and\ \overline{r}^- = (0, 1)$$

Using the Euclidean distance function, we can find the distances between the collective overall performance values$\overline{r}_i$, and the positive ideal vague value$\overline{r}^+$as follows:

$$d(\overline{r}_i, \overline{r}^+) = \sqrt{\frac{1}{2}\sum_{i=1}^{m}\left[\left(t_{\overline{r}_i}(x_i) - t_{\overline{r}^+}(x_i)\right)^2 +; \left(\left(1 - f_{\overline{r}_i}(x_i)\right) - \left(1 - f_{\overline{r}^+}(x_i)\right)\right)^2\right]}$$

Step – 4:

Rank all the alternatives $A_i, for\ i = 1, \dots, m$ and select the best one in accordance with the distance obtained in Step – 3.

3.22. Example – Numerical Illustration

Suppose an investment company, wanting to invest a sum of money in the best option, and there is a panel with five possible alternatives to invest the money.

A_1, it is IT Company

A_2, it is a multinational Company

A_3, it is a tool Company

A_4, it is an Airline Company

A_5, it is an Automobile Company

The investment company must take a decision according to the four following attributes

G_1, it is the risk analysis, G_2, it is the growth analysis; G_3, it is the socio – political impact analysis; G_4, it is the environmental impact analysis

3.23. The five possible alternatives $A_i, for\ i = 1,2,3,4,5$, they are to be evaluated by three decision makers, their weighting vector $V = [0.35, \quad 0.40, \quad 0.25]^T$ under the above said four attributes whose weighting vector is $\omega = [0.2, \quad 0.1, \quad 0.3, \quad 0.4]^T$, which gives the decision matrices of vague values $R_k = \left(\bar{r}_{ij}^{(k)}\right)_{5\times4}, for\ k = 1,2,3$

Solution:

$$R_1 = \begin{bmatrix} (0.4427, & 0.9986) & (0.6286, & 0.8312) & (0.5221, & 0.7222) & (0.4873, & 0.7256) \\ (0.7710, & 0.9442) & (0.4261, & 0.8126) & (0.6676, & 0.5413) & (0.3271, & 0.9001) \\ (0.5687, & 0.7981) & (0.5527, & 0.6216) & (0.4278, & 0.5261) & (0.5238, & 0.8011) \\ (0.6626, & 0.8215) & (0.8311, & 0.9219) & (0.7213, & 0.8912) & (0.7218, & 0.6283) \\ (0.4136, & 0.6295) & (0.6256, & 0.7119) & (0.8321, & 0.9426) & (0.4351, & 0.7983) \end{bmatrix}$$

$$R_2 = \begin{bmatrix} (0.2217, & 0.7184) & (0.1009, & 0.6221) & (0.5121, & 0.7221) & (0.4351, & 0.7846) \\ (0.6225, & 0.9105) & (0.3009, & 0.5129) & (0.6226, & 0.8108) & (0.6321, & 0.8221) \\ (0.4991, & 0.5426) & (0.5010 & 0.7101) & (0.4124, & 0.7216) & (0.5387, & 0.9105) \\ (0.2101, & 0.4110) & (0.2091, & 0.4104) & (0.5221, & 0.8001) & (0.7317, & 0.8119) \\ (0.6210, & 0.8109) & (0.4728, & 0.7182) & (0.3125, & 0.7278) & (0.5273, & 0.6217) \end{bmatrix}$$

$$R_3 = \begin{bmatrix} (0.6661, & 0.7027) & (0.2211, & 0.5221) & (0.4419, & 0.9816) & (0.3198, & 0.8279) \\ (0.7101, & 0.9005) & (0.6216, & 0.8225) & (0.6245, & 0.7815) & (0.7726, & 0.8901) \\ (0.6105, & 0.9117) & (0.7117, & 0.9211) & (0.5821, & 0.6286) & (0.5201, & 0.7287) \\ (0.5529, & 07217) & (0.4212, & 05334) & (0.7139, & 0.8148) & (0.3247, & 0.4821) \\ (0.4214, & 0.5005) & (0.2221, & 0.6121) & (0.8001, & 0.9112) & (0.7351, & 0.9113) \end{bmatrix}$$

Explanations of the above result by step wise as above given as follows:

Step – 1:

Utilizing the decision information given in the matrix $R_k = \left(\bar{r}_{ij}^{(k)}\right)_{5\times4}\ for\ k = 1,2,3$, and the IVOWA operator which has the associated weighting vector $\omega = [0.2, \quad 0.1, \quad 0.3, \quad 0.4]^T$, we get collective decision matrix $R_k = \left(\bar{r}_{ij}^{(k)}\right)_{5\times4}$

$$R = \begin{bmatrix} (0.5020, & 0.8267) & (0.4256, & 0.6666) & (0.4933, & 0.8041) & (0.4229, & 0.7718) \\ (0.7252, & 0.9219) & (0.4540, & 0.7443) & (0.6442, & 0.6673) & (0.5921, & 0.8805) \\ (0.5629, & 0.7741) & (0.6080, & 0.7326) & (0.4847, & 0.5964) & (0.5255, & 0.7951) \\ (0.5586, & 0.6835) & (0.6461, & 0.6475) & (0.6868, & 0.8452) & (0.6233, & 0.6028) \\ (0.4651, & 0.6111) & (0.4821, & 0.6764) & (0.7634, & 0.8845) & (0.6525, & 0.7954) \end{bmatrix}$$

Step – 2:

Utilizing the VWA operator, we obtain the collective overall performance values of the alternatives $A_i, for\ i = 1,2,3,4,5$

$\overline{r}_1 = (0.4614, \quad 0.2143); \overline{r}_2 = (0.6275, \quad 0.1610); \overline{r}_3 = (0.5306, \quad 0.2630)$

$\overline{r}_4 = (0.6460, \quad 0.2827)\ and\ \overline{r}_5 = (0.6487, \quad 0.2052)$

Step – 3:

Calculating the distance between the collective performance values$\overline{r}_i$, and the positive ideal vague value$\overline{r}^+ = (1, 0)$, the distances calculated from the following distance function

$$d(\overline{r}_i, \overline{r}^+) = \sqrt{\frac{1}{2}\sum_{i=1}^{4}\left[\left(t_{\overline{r}_i}(x_i) - t_{\overline{r}^+}(x_i)\right)^2 + \left(\left(1 - f_{\overline{r}_i}(x_i)\right) - \left(1 - f_{\overline{r}^+}(x_i)\right)\right)^2\right]}$$

That is we have

$d(\overline{r}_1, \overline{r}^+) = 0.409965 \approx 0.4100$

$d(\overline{r}_2, \overline{r}^+) \approx 2869$

$d(\overline{r}_3, \overline{r}^+) \approx 0.3805$

$d(\overline{r}_4, \overline{r}^+) \approx 0.3203$

$d(\overline{r}_5, \overline{r}^+) = 0.2877$

Step – 4:

Rank the alternatives based on the shortest distance:

$A_2 < A_5 < A_4 < A_3 < A_1$

Result:

Hence A_2, it is the best alternative

4. CONCLUSION

Vague set theory has received more and more attention, because may the real life problems information's are in the form of vague values. For Multiple Attribute Group Decision Making (MAGDM) problems where the attribute weights and the expert weights are real numbers and the attribute values take the form of vague values. In this article the new operator (IVOWA) is utilized for aggregating the vague information. The induced vague ordered weighted averaging operator (IVOWA) for vague sets is applied in MAGDM model and also it is develop0ed based on VOWA operator and the vague weighted averaging (VWA) operator. A simple reasonable numerical illustration is presented to show the effectiveness of the proposed mode and comparison of the proposed model is made with an existing method.

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