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# SQUARE SUM PRIME LABELING OF SOME CYCLE RELATED GRAPHS

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**Abstract:** *Square sum prime labeling of a graph is the labeling of the vertices with {0, 1, 2-------, p-1} and the edges with sum of the squares of the labels of the incident vertices. The greatest common incidence number of a vertex (**gcin**) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the **gcin** of each vertex of degree greater than one is one, then the graph admits square sum prime labeling. Here we identify some cycle related graphs for square sum prime labeling.*

***Keywords:*** *Graph labeling, square sum, greatest common incidence number, prime labeling.*

## 1. INTRODUCTION

All graphs in this paper are simple, finite, connected and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)-graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1] and [2]. The square sum labeling was defined by V Ajitha, S Arumugan and K A Germina in [5]. In this paper we introduced square sum prime labeling using the concept greatest common incidence number of a vertex. We proved that some cycle related graphs admit square sum prime labeling.

**Definition: 1.1** Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

## 2. MAIN RESULTS

**Definition 2.1** Let G = (V, E) be a graph with p vertices and q edges. Define a bijection

f : V(G) → {0,1,2,3,----------------,p-1} by $f(v_i) = i-1$ , for every i from 1 to p and define a 1-1 mapping $f_{sqsp}^{*}$ : E(G) → set of natural numbers N by $f_{sqsp}^{*}(uv) = \{f(u)\}^2 + \{f(v)\}^2$ . The induced function $f_{sqsp}^{*}$ is said to be a sum square prime labeling, if for each vertex of degree at least 2, the greatest common incidence number is 1.

**Definition 2.2** A graph which admits square sum prime labeling is called a square sum prime graph.

**Theorem 2.1** Cycle $C_n$ admits square sum prime labeling, when n is odd.

Proof: Let G = $P_n$ and let $v_1,v_2$,----------------,$v_n$ are the vertices of G

Here |V(G)| = n and |E(G)| = n

Define a function f : V → {0,1,2,3,----------------,n-1 } by

$f(v_i)$ = i-1 , i = 1,2,------,n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f_{sqsp}^{*}$ is defined as follows

$f_{sqsp}^{*}(v_i\, v_{i+1})$ = $2i^2 - 2i + 1$, i = 1,2,------------,n-1

$f_{sqsp}^{*}(v_1\, v_n)$ = $n^2 - 2n + 1$.

Clearly $f_{sqsp}^{*}$ is an injection.

***gcin*** of ($v_{i+1}$) = gcd of $\{f_{sqsp}^{*}(v_i\, v_{i+1}), f_{sqsp}^{*}(v_{i+1}\, v_{i+2})\}$

= gcd of { $2i^2+2i+1$, $2i^2-2i+1$}

= gcd of {4i, $2i^2-2i+1$},

= gcd of {i, $2i^2-2i+1$} = 1, i = 1,2,------------,n-2

***gcin*** of ($v_1$) = gcd of $\{f_{sqsp}^{*}(v_1\, v_2), f_{sqsp}^{*}(v_1\, v_n)\}$

= gcd of { 1, $(n-1)^2$}= 1

***gcin*** of ($v_n$) = gcd of $\{f_{sqsp}^{*}(v_1\, v_n), f_{sqsp}^{*}(v_{n-1}\, v_n)\}$,

= gcd of { $2n^2-6n+5$, $n^2-2n+1$},

= gcd of { $n^2-4n+4$, $n^2-2n+1$}= g c d of {n-2,n-1} = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $C_n$, admits square sum prime labeling. ■

**Theorem 2.2** Middle graph of cycle $C_n$ admits square sum prime labeling, when n is an even integer.

Proof: Let G = M($C_n$) and let $v_1,v_2$,----------------,$v_{2n}$ are the vertices of G

Here |V(G)| = 2n and |E(G)| = 3n

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

$f(v_i)$ = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f_{sqsp}^*$ is defined as follows

$f_{sqsp}^*(v_i\ v_{i+1})$ = $2i^2 - 2i + 1$, i = 1,2,------------,2n-1

$f_{sqsp}^*(v_{2i}\ v_{2i+2})$ = $8i^2+2$ i = 1,2,------------,n-1

$f_{sqsp}^*(v_1\ v_{2n})$ = $(2n-1)^2$.

$f_{sqsp}^*(v_2\ v_{2n})$ = $(2n-1)^2 + 1$.

Clearly $f_{sqsp}^*$ is an injection.

***gcin*** of ($v_{i+1}$) = 1, i = 1,2,------------,2n-2

***gcin*** of ($v_1$) = 1.

***gcin*** of ($v_{2n}$) = gcd of $\{f_{sqsp}^*(v_1\ v_{2n}), f_{sqsp}^*(v_{2n-1}\ v_{2n})\}$

= gcd of $\{(2n-1)^2, 8n^2-12n+5\}$

= gcd of {2n-1, (2n-1)(4n-4)+1}

=1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence M($C_n$), admits square sum prime labeling. ■

**Theorem 2.3** Total graph of cycle $C_n$ admits square sum prime labeling, when is even.

**Proof:** Let G = T($C_n$) and let $v_1, v_2$,----------------,$v_{2n}$ are the vertices of G

Here |V(G)| = 2n and |E(G)| = 4n.

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

$f(v_i) = i-1$ , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f_{sqsp}^*$ is defined as follows

$f_{sqsp}^*(v_i\ v_{i+1})$ = $2i^2-2i+1$, i = 1,2,------------,2n-1

$f_{sqsp}^*(v_{2i}\ v_{2i+2})$ = $8i^2+2$, i = 1,2,------------,n-1

$f_{sqsp}^*(v_{2i-1}\ v_{2i+1})$ = $8i^2-8i+4$, i = 1,2,------------,n-1

$f_{sqsp}^*(v_1\ v_{2n})$ = $4n^2-4n+1$.

$f_{sqsp}^*(v_2\ v_{2n})$ = $4n^2-4n+2$.

$f_{sqsp}^*(v_1\ v_{2n-1})$ = $4n^2-8n+4$.

Clearly $f_{sqsp}^*$ is an injection.

***gcin*** of ($v_{i+1}$) = 1, i = 1,2,------------,2n-2

***gcin*** of ($v_1$) = 1.

***gcin*** of ($v_{2n}$) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence T($C_n$), admits square sum prime labeling. ■

**Theorem 2.4** Corona of cycle $C_n$ admits square sum prime labeling.

Proof: Let G = $C_n \odot K_1$ and let $v_1,v_2$,----------------,$v_{2n}$ are the vertices of G

Here |V(G)| = 2n and |E(G)| = 2n

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

$f(v_i)$ = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsp}$ is defined as follows

**Case(i) n is even**

$f^*_{sqsp}(v_i\, v_{i+1})$ = $2i^2$-2i+1, i = 1,2,------------,n+1

$f^*_{sqsp}(v_2\, v_{n+1})$ = $n^2$+1,

$f^*_{sqsp}(v_{i+2}\, v_{n+i+2})$ = $n^2+2n(i+1)+2(i+1)^2$, i = 1,2,------------,n-2

Clearly $f^*_{sqsp}$ is an injection.

***gcin*** of ($v_{i+1}$) = 1, i = 1,2,------------,n

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $C_n \odot K_1$ admits square sum prime labeling, when n is even.

**Case(ii) n is odd**

$f^*_{sqsp}(v_i\, v_{i+1})$ = $2i^2$-2i+1, i = 1,2,------------,n-1

$f^*_{sqsp}(v_1\, v_n)$ = $n^2$-2n+1,

$f^*_{sqsp}(v_i\, v_{2n-i+1})$ = $4n^2-4ni+2i^2-2i+1$, i = 1,2,------------,n

Clearly $f^*_{sqsp}$ is an injection.

***gcin*** of ($v_{i+1}$) = 1, i = 1,2,------------,n-1

***gcin*** of ($v_1$) = gcd of { 1,$(n-1)^2$} = 1,

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $C_n \odot K_1$ admits square sum prime labeling, when n is odd.

**Theorem 2.5** Wheel graph $W_n$ admits square sum prime labeling, when n is odd.

Proof: Let G = $W_n$ and let u,$v_1,v_2$,----------------,$v_n$ are the vertices of G

Here |V(G)| = n+1 and |E(G)| = 2n

Define a function f : V → {0,1,2,3,----------------,n } by

$f(v_i)$ = i , i = 1,2,------,n

f(u) = 0

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f_{sqsp}^*$ is defined as follows

$f_{sqsp}^*(u\ v_i)$ = $i^2$, i = 1,2,------------,n.

$f_{sqsp}^*(v_1\ v_n)$ = $n^2+1$,

$f_{sqsp}^*(v_i\ v_{i+1})$ = $2i^2+2i+1$, i = 1,2,------------,n-1

Clearly $f_{sqsp}^*$ is an injection.

***gcin*** of (u) = 1,

***gcin*** of ($v_{i+1}$) = 1, i = 1,2,------------,n-1

***gcin*** of ($v_n$) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $W_n$ admits square sum prime labeling, when n is odd.

**Theorem 2.6** $C_n(P_m)$ admits square sum prime labeling, when n is odd.

Proof: Let G = $C_n(P_m)$ and let $v_1,v_2$,----------------,$v_{n+m-1}$ are the vertices of G

Here |V(G)| = n+m-1 and |E(G)| = n+m-1

Define a function f : V → {0,1,2,3,----------------,n+m-2 } by

$f(v_i)$ = i-1 , i = 1,2,------,n+m-1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f_{sqsp}^*$ is defined as follows

$f_{sqsp}^*(v_i\ v_{i+1})$ = $2i^2-2i+1$, i = 1,2,------------,n+m-2

$f_{sqsp}^*(v_1\ v_n)$ = $n^2-2n+1$,

Clearly $f_{sqsp}^*$ is an injection.

***gcin*** of ($v_{i+1}$) = 1, i = 1,2,------------,n+m-3

***gcin*** of ($v_1$) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $C_n(P_m)$ admits square sum prime labeling, when n is odd.

**Theorem 2.7** $C_n(2P_m)$ admits square sum prime labeling, when n, m odd.

Proof: Let G = $C_n(2P_m)$ and let $v_1,v_2$,----------------,$v_{n+2m-2}$ are the vertices of G

Here |V(G)| = n+2m-2 and |E(G)| = n+2m-2

Define a function f : V → {0,1,2,3,----------------,n+2m-3 } by

$f(v_i)$ = i-1 , i = 1,2,------,n+2m-2

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f_{sqsp}^*$ is defined as follows

$f_{sqsp}^*(v_i\ v_{i+1})$ = $2i^2-2i+1$, i = 1,2,------------,n+2m-3

$f^*_{sqsp}(v_m\ v_{m+n-1})$ = $(m-1)^2 + (m+n-2)^2$

Clearly $f^*_{sqsp}$ is an injection.

***gcin*** of ($v_{i+1}$) = 1, i = 1,2,------------,n+2m-4

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $C_n(2P_m)$ admits square sum prime labeling, when n is odd.

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