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## GPS ELLIPSOID HEIGHT CALIBRATION TO DETERMINE THE APPROXIMATE MEAN SEA LEVEL (ORTHOMETRIC) HEIGHT

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**Abstract:** *Through this paper a comparison between the ellipsoidal GPS heights and the Digital Level had been accomplished to have a constant value between the mathematical (ellipsoid) model of the earth and the physical (geoid) shape of the earth, intending to get approximately the constant separation between the geoid and the ellipsoid. Over the considered area common points with GPS and leveling were measured by leveling instruments and GPS Real Time receivers were used to get the height separation between the geoid ( M.S.L.) and the ellipsoid, and then the constant separation had been used as the height transformation parameters of the GPS height to a leveling height. Results obtained showed that the transformed (calibrated) GPS has an accurate and a consistent height readings which fit to the leveling heights on the common points of a large scale map of an engineering plane surface project which do not need a very accurate height determination.*

**Keywords:** *DGPS, Differential leveling, Precise leveling, Geoid, Orthometric height.*

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## **1. INTRODUCTION**

The elevation of a point near the surface of the earth is its vertical distance above or below an arbitrary assumed level surface or curved surface, every element of which is normal to the plumb line. The level surface (real or imaginary) used for reference is called the datum. A level line is a line in a level surface. The difference in elevation between two points is the vertical distance between the two level surfaces in which the point lie. Leveling is the operation of measuring vertical distance, either directly or indirectly, to determine differences in elevation.

The datum most commonly used is mean sea level, particularly the Mean Sea Level Datum established in Egypt at Alexandria for Egypt and Sudan Leveling datum. Another datum for height determination is the surface of the ellipsoid. It is a mathematical surface which does not fit to geoid which is approximately mean sea level surface. The height above the geoid is measured by the plumb line, which is vertical to the geoid, while the height above the ellipsoid, which is measured by the GPS, is normal to the ellipsoid. Then the vertical plumb line and the normal ellipsoidal height are deviated by small deviation angle indicating separation between the surface of the geoid and the ellipsoid.

The Mean Sea Level (M.S.L.) is the approximate surface which fit to the surface of the geoid. Then surface of the geoid is the reference surface for the useful height determination of any point on the topography of the earth. The geoid (approximately M.S.L.) height is the only height which can be used for water flow contour elevations. Leveling is the usual technique used to determine difference in height between ground points which guide to the contour lines and the intervals of the water flow on the surface of the topography. The Global Positioning System (GPS) is a quick modern technology to determine the earth surface heights above the ellipsoid but this height can not be confidently determine difference in height between ground points which can be used to determine the topographic contour map. Separation between ellipsoid and geoid height can transform the ellipsoid height measured by GPS to mean sea level (approximately the geoid) height useful for contour maps.

## 2. LITERTURE REVIEW

### 2.1 THE GPS REAL TIME SYSTEM

The introduction of the Global Positioning System (GPS) Satellite navigation system promised to give the instantaneous positioning capability to the navigation community [4] . After all GPS is designed to provide accurate navigation World – Wide , 24 hours a day . But the meaning of accurate to the navigation crowd is quite different to what is considered accurate to land surveyor, but if you are surveying a subdivision that is considered a gross error , at best. Through a great deal of survey measurements research has focused on the use of GPS and, subsequently, many breakthroughs have been made. Centimeter and even millimeter level accuracy became common place . Now, GPS Differential Real Time Kinematic, surveying gives us the capability walk from point to point and see where they are while they are moving (see Fig. 1) . Using both GPS radio-modem technology, GPS Differential Real Time Kinematic produces the exact coordinate of the point while you are occupying it. That means that you can survey points by simply walking to them . But , more than that, if you can get accurate coordinates at any time during the survey, you can use that information to guide you where you need to be. In sense, GPD Differential Real Time Kinematic lets you accurately navigate to traverse points, as well as survey them . Sometimes, than you imagined possible.

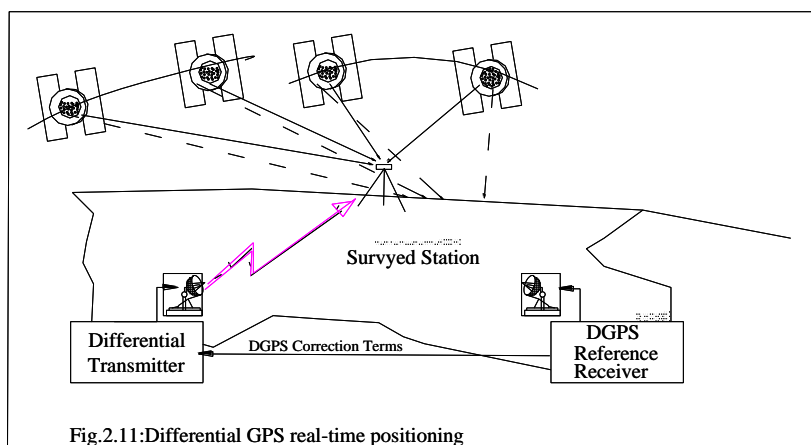


Fig.1: Differential GPS Real Time Positioning System

### 2.2 VERTICAL CONTROL

Precise leveling is used for establishing the elevations of the controlling bench marks while serve as starting points for all subsequent leveling.

### 2.2.1 PRECISE LEVELING

Precise spirit leveling differs from ordinary leveling in that certain refinements are introduced into the methods of procedure and into the design of the instrument. Lines of Precise levels have been run by the Sudan Survey Department since beginning of twenty's century covering many areas of the country. Recently, The Sudan Dam Implementation Unit (DIU) had established first order leveling lines persuading the Nile River bank from Halfa at the Northern Sudan to Kosti on the bank of the White Nile [2].

### 2.3 DIFFERENTIAL ORDINARY LEVELING

Differential leveling is adopted when:

i) the points are a great distance apart, ii) the difference of elevation between the points is large, ii) there are obstacles between the points.

This method is also known as compound leveling. In this method, the ordinary level is set up at several suitable points and staff readings are taken at back and foresight, while the level has to be set up at the middle point between the back and the foresight locations consider figure 2. Suppose it is required to know the difference of level between A and B, the level is set up on points  $S_1, S_2, S_n$  etc. After temporary adjustments, staff readings are taken at every set up. The points  $I_1, I_2$  and  $I_{n-1}$  are known as change points (see Fig. 2). Then the difference of level between A and B is found out. If the difference is positive, A is lower than B. If it is negative, A is higher than B. Knowing the Reduced Level (RL) of A and that of B can be calculated [1].

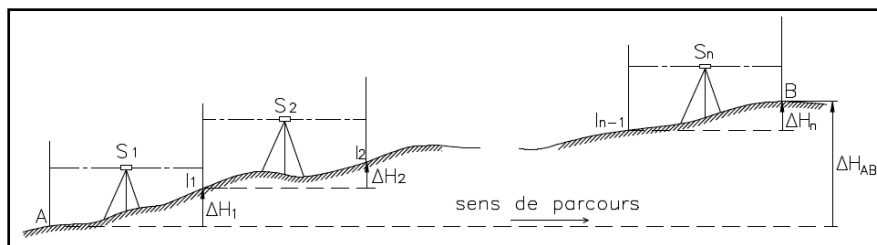


Fig. 2: longitudinal leveling line.

## 3. METHODOLOGY

### 3.1 Determination of the mean sea level (orthometric) height (H):

After the calibration and adjustment of the Digital Level, the leveling research project was began, by dividing the proposed points which has to be leveled, according to the number of the Bench Marks (B.M.) into three groups. Each group was leveled independently and the



readings were processed to determine the Reduced level with an acceptable closing error. Table(1) showed the heights (H) of 17 points reduced from the level readings.

### **3.2 Determination of the ellipsoid heights (GPS heights (h)):**

After finishing from the leveling and reducing the orthometric heights of the above points explained in table (1), the Real Time GPS observation had been accomplished on the same points in table (1). After processing and the adjustment computation of the Differential GPS Real Time, observations, three dimension positioning coordinates Northing, Easting and Ellipsoid height (h) were retrieved, see table (1).

### **3.3 Orthometric height (H) and Ellipsoid heights (h):**

Orthometric height (H) elevations are not to be confused with GPS heights (h). GPS heights are based on a mathematical surface called the ellipsoid, while elevations are based on a level surface called the geoid (approximately mean sea level). Both of these surfaces are the zero points for their respective heights. To convert the GPS ellipsoid height to an orthometric height elevation, it is very important to know the difference in height between the two systems; this difference may not be constant. The differences may change gradually from point to point in any given area. These changes are called the geoidal undulations.

## **4. RESULTS AND ANALYSIS**

The 17 points were observed commonly with the digital level for mean sea Level H (orthometric) height and with the Differential GPS Real Time geodetic receivers to get the WGS84 ellipsoid height h , which have been shown together in table (1). Table (1) also showed the difference between WGS84 ellipsoid height (h) and the mean sea level(orthometric) height (H), ( $h-H=N$ ), beside the mean of the differences (h-H). Table (2) showed the reduced ellipsoid height to mean sea level (orthometric) height (H') using the mean of the geoid ellipsoid separation to calibrate ellipsoid heights of the seventeen points to geoid (orthometric) height. Table (3) showed the calibrated ellipsoid heights using the geoid ellipsoid separation by the mean of two points (points 1 and 2). Table (4) showed the calibrated ellipsoid heights using the geoid ellipsoid separation by the mean of three points (points 1,2 and 3). Table (5) showed the calibrated ellipsoid heights using the geoid ellipsoid separation by the mean of other three points (points 4, 8 and 14). Each reduction in each table has its own mean and standard deviation.



At the final stage it is useful to check the above results by using the Differential Geodetic GPS Real Time Kinematic receivers. Then accordingly point one was choosed as the reference station (base station) and the horizontal coordinates (E , N) was given to the base station (point 1) reciver but for the elevation, the base station receiver was given the value  $H'$  in table(2) which is equal to the mean of the geoid ellipsoid separation ( $N_{mean}$ ) subtracted from the ellipsoid height  $h_1$  of point 1, ( $H' = h_1 - N_{mean}$ ) see table (2) in which  $H' = 388.12$  meters. After that the rover receiver was stationed and centered on each of the other 16 points (starting with point two and finishing with point 17) to get the calibrated orthometric height ( $H'$  of table 6) directly by GPS Real Time Kinematic observation see table (6). Then the difference between the actual orthometric height ( $H$  as in tale 2 ) and GPS calibrated height ( $H'$  of table 6) which was determined ( see table 6 ). Finally the standard deviation of the differences was obtained (see table 6), the value of the standard deviation is equal to 0.009807 meters.

## 5. CONCLUSIONS AND RECOMMENATIONS

### 5.1 CONCLUSIONS

After computing the field observation data obtained by the digital level and the differential real time GPS receivers, results computed showed that the mean ( ~~$N_{mean}$~~ ) of the geoid ellipsoid separation is equal to 6.84402 meters .

The case study summarized and concluded that the standard deviation computed for the difference between the actual orthometric heights and the computed orthometric height using the mean of the ellipsoid and geoid separation for the first point (point 1), first two points (point 1 and 2), first three points (1,2 and 3) and the other three points (point 4,8, and 14) has the same value which is equal to 0.008187 meters and accordingly this conclusion is realized that ellipsoid and geoid separation of one of any 17 points is enough to be used for the calibration of the differential real time GPS receivers to get the orthometric height which will obtained by the differential GPS receivers for unknown points covering the considered area (area for any engineering project (scheme)).

Results were showed that the greatest difference between the actual orthometric heights and the calibrated differential GPS calibrated heights (see table 6) is equal to -0.02289 meters and the standard deviation of the differences is equal to 0.009807 meters .



Finally it could be concluded that the actual orthometric height determined by the digital level observation is more accurate than the orthometric height determined by the calibrated differential real time GPS.

The orthometric heights determined by the calibrated differential real time GPS observation are useful to be used in the projects (scheme) which has no need very accurate height values just like the mapping projects of engineering schemes (maps for studying the area of dams implementation, bridges, railways, highways ..etc.).

## **5.2 RECOMMENDATIONS**

Digital and ordinary spirit levels are more accurate to be used for determination of the reference heights (Bench Mark values) but the calibrated differential real time GPS observation heights can be used in the projects which has no need for very accurate heights. The ground area of this research is approximately has a plane surface with no irregular undulations over the area and then it is recommended to have another similar investigation in the future over irregular undulated and hilly area.

## **ACKNOWLEDGEMENTS**

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## **REFERENCES**

- [1] N. N. BASAK, Surveying and Leveling, by, Department of civil Engineering , Malada polytechnic , Malada-West Bengal) .
- [2] Paul R. Wolf and Charlis D. Ghilani, (2006), Elementary Surveying, 11th Edition, Pearson prentice hall, New Jersey.
- [3] William Irvine, (1995) Surveying for Construction, 4th Edition, Macmillan education Ltd. Sokkia power set SDR software reference manual, (2001), USA.
- [4] Surveying with GPS Training manual (1996), Trimble navigation limited, USA.
- [5] [http:// earthinfo.nima.mil/ gandg/ wgs84/gravitymod/egm2008](http://earthinfo.nima.mil/gandg/wgs84/gravitymod/egm2008).
- [6] [http:// earthinfo.nima.mil/ gandg/ wgs84/ gravitymod/ index.html](http://earthinfo.nima.mil/gandg/wgs84/gravitymod/index.html)
- [7] [http://earthinfo.nima.mil/gandg/wgs84/gravitymod/new\\_egm -](http://earthinfo.nima.mil/gandg/wgs84/gravitymod/new_egm)
- [8] [http:// icgem. gfz Potsdam. de/ icgem/ icgem.html](http://icgem.gfz Potsdam.de/icgem/icgem.html)
- [9] [http:// gomaa.dawod.googlepages. com/geoidofegypt](http://gomaa.dawod.googlepages.com/geoidofegypt)



[10] <http://icgem.gfz Potsdam.de/icgem/shms/egm2008.gfcinfo.nima.mil/gandg/wgs84/gravitymod/egm2008>

Table(1) :The geoid and ellipsoid separation  $H$  (Orthometric Height) - $h$  (Ellipsoid height ) for each of 17 points.

Index	Easting m	Northing m	Orthometric Height ( $H$ ) m	Ellipsoid height ( $h$ )	Ellipsoid Geoid separation ( $h-H$ )= $N$
1	448972.7	1745043	388.1189	394.964	6.84515
2	449141.9	1745016	386.1158	392.968	6.8522
3	449251.8	1744930	385.3717	392.223	6.85135
4	449335.3	1744912	383.1992	390.059	6.8598
5	449362.9	1745036	382.8722	389.725	6.85285
6	449278.5	1744526	382.7898	389.622	6.83225
7	449303.8	1744667	384.0386	390.887	6.8484
8	449184.8	1744711	385.8333	392.666	6.83275
9	449123	1744847	389.0224	395.87	6.8476
10	448978.2	1744777	386.7984	393.642	6.84365
11	449341.2	1744412	382.7749	389.608	6.8331125
12	449485.3	1744394	381.2648	388.1	6.835225
13	449573	1744658	381.7204	388.561	6.8406375
14	449604.2	1744790	380.7434	387.585	6.84165
15	449614.7	1744892	379.5744	386.426	6.8515625
16	449787.3	1744715	380.2144	387.051	6.836575
17	449812	1744884	379.0444	385.888	6.8435875
Sum					116.3484
Mean					6.844021

Table(2) The mean of the (geoid and ellipsoid) separation  $N$  of the 17 points was used to get the computed and calibrated (orthometric height)  $H'$ , then the standard deviation of  $H'-H$  for the 17 points was computed .

point	$H$ m	$H$ m	$N$	$H'$	$H'-H$
1	394.964	388.1189	6.84515	388.12	0.001129
2	392.968	386.1158	6.8522	386.124	0.008179
3	392.223	385.3717	6.85135	385.379	0.007329
4	390.059	383.1992	6.8598	383.215	0.015779
5	389.725	382.8722	6.85285	382.881	0.008829
6	389.622	382.7898	6.83225	382.778	-0.01177
7	390.887	384.0386	6.8484	384.043	0.004379
8	392.666	385.8333	6.83275	385.822	-0.01127
9	395.87	389.0224	6.8476	389.026	0.003579





10	393.642	386.7984	6.84365	386.798	-0.00037
11	389.608	382.7749	6.833113	382.764	-0.01091
12	388.1	381.2648	6.835225	381.256	-0.0088
13	388.561	381.7204	6.840637	381.717	-0.00338
14	387.585	380.7434	6.84165	380.741	-0.00237
15	386.426	379.5744	6.851563	379.582	0.007542
16	387.051	380.2144	6.836575	380.207	-0.00745
The mean			6.844021		
STDEV					0.008187

Table(3):The mean of the (geoid and ellipsoid) separation of two points (points1 and 2) was used to get the computed and calibrated orthometric height)  $H'$ , then the standard deviation of  $H'-H$  for the 17 points was computed .

point	h m	H m	N	$H'$	$H'-H$
1	394.964	388.1189	6.84515	388.1153	-0.00353
2	392.968	386.1158	6.8522	386.1193	0.003525
3	392.223	385.3717		385.3743	0.002675
4	390.059	383.1992		383.2103	0.011125
5	389.725	382.8722		382.8763	0.004175
6	389.622	382.7898		382.7733	-0.01643
7	390.887	384.0386		384.0383	-0.00028
8	392.666	385.8333		385.8173	-0.01593
9	395.87	389.0224		389.0213	-0.00108
10	393.642	386.7984		386.7933	-0.00503
11	389.608	382.7749		382.7593	-0.01556
12	388.1	381.2648		381.2513	-0.01345
13	388.561	381.7204		381.7123	-0.00804
14	387.585	380.7434		380.7363	-0.00703
15	386.426	379.5744		379.5773	0.002888
16	387.051	380.2144		380.2023	-0.0121
17	385.888	379.0444		379.0393	-0.00509
Mean			6.848675		
ST.Div.					0.008187



Table (4):The mean of the (geoid and ellipsoid) separation of three points (points1 , 2 and 3) was used to get the computed and calibrated orthometric height)  $H'$ , then the standard deviation of  $H'-H$  for the 17 points was computed .

point	H m	H m	N	$H'$	$H'-H$
1	394.964	388.1189	6.84515	388.1144	-0.00442
2	392.968	386.1158	6.85220	386.1184	0.002633
3	392.223	385.3717	6.85135	385.3734	0.001783
4	390.059	383.1992		383.2094	0.010233
5	389.725	382.8722		382.8754	0.003283
6	389.622	382.7898		382.7724	-0.01732
7	390.887	384.0386		384.0374	-0.00117
8	392.666	385.8333		385.8164	-0.01682
9	395.87	389.0224		389.0204	-0.00197
10	393.642	386.7984		386.7924	-0.00592
11	389.608	382.7749		382.7584	-0.01645
12	388.1	381.2648		381.2504	-0.01434
13	388.561	381.7204		381.7114	-0.00893
14	387.585	380.7434		380.7354	-0.00792
15	386.426	379.5744		379.5764	0.001996
16	387.051	380.2144		380.2014	-0.01299
17	385.888	379.0444		379.0384	-0.00598
Mean			6.849567		
ST.DEV.					0.008187

Table(5):The mean of the (geoid and ellipsoid) separation of other three points (points 4 ,8 and 14) was used to get the computed and calibrated orthometric height)  $H'$ , then the standard deviation of  $H'-H$  for the 17 points was computed .

point	H m	H m	N	$H'$	$H'-H$
1	394.964	388.1189		388.1193	0.000417
2	392.968	386.1158		386.1233	0.007467
3	392.223	385.3717		385.3783	0.006617
4	390.059	383.1992	6.8598	383.2143	0.015067
5	389.725	382.8722		382.8803	0.008117
6	389.622	382.7898		382.7773	-0.01248
7	390.887	384.0386		384.0423	0.003667
8	392.666	385.8333	6.83275	385.8213	-0.01198
9	395.87	389.0224		389.0253	0.002867
10	393.642	386.7984		386.7973	-0.00108
11	389.608	382.7749		382.7633	-0.01162
12	388.1	381.2648		381.2553	-0.00951



13	388.561	381.7204		381.7163	-0.0041
14	387.585	380.7434	6.84165	380.7403	-0.00308
15	386.426	379.5744		379.5813	0.006829
16	387.051	380.2144		380.2063	-0.00816
17	385.888	379.0444		379.0433	-0.00115
Mean			6.844733		
ST.DIV.					0.008187

Table(6): The GPS calibrated orthometric heights for each of 17 points by giving the GPSRTK receiver, the computed orthometric height  $H'$  of point 1 shown on table (5) .

point	$H'$ m	H m	$H'-H$
1	388.12	388.1189	0.00115
2	386.117	386.1158	0.0012
3	385.354	385.3717	-0.01765
4	383.185	383.1992	-0.0142
5	382.891	382.8722	0.01885
6	382.786	382.7898	-0.00375
7	384.033	384.0386	-0.0056
8	385.831	385.8333	-0.00225
9	389.025	389.0224	0.0026
10	386.794	386.7984	-0.00435
11	382.752	382.7749	-0.02289
12	381.262	381.2648	-0.00278
13	381.722	381.7204	0.001637
14	380.754	380.7434	0.01065
15	379.577	379.5744	0.002563
16	380.209	380.2144	-0.00542
17	379.045	379.0444	0.000588
ST.DIV.			0.009807